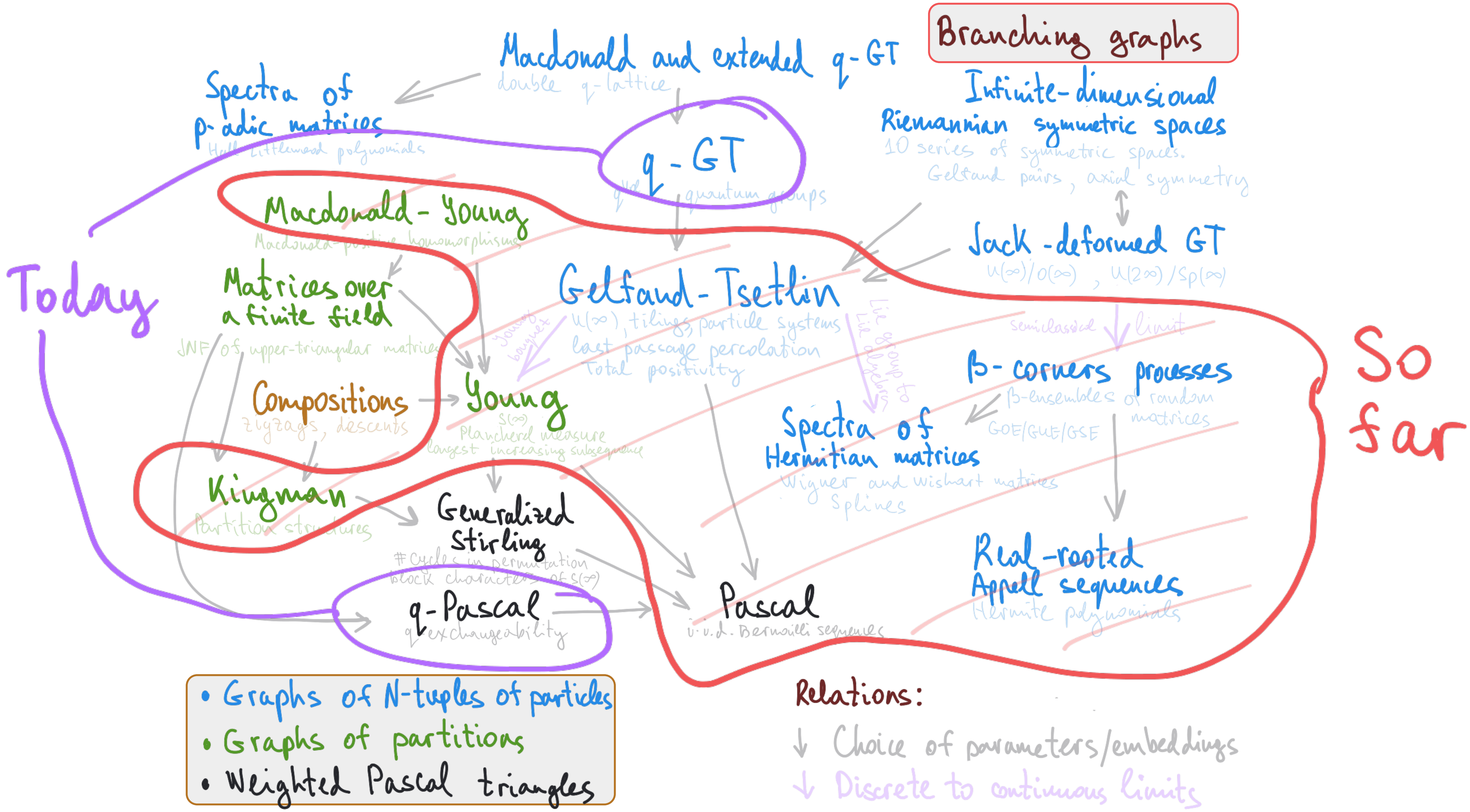
Math 833 q-deformations Week 12



So far: while the graphs were both discrete and continuous, but all the boundaries were continuous.

Today we study quantizations. Some meanings:

- Continuous set $\longrightarrow$ discrete approximation
- Deformation of weights $1 \longrightarrow q^{v}$

  some function of path

  quantization parameter
- Deformation of objects: sets $\longrightarrow$ subspaces

  groups $\longrightarrow$ quantum groups

We start from q-Pascal graph.

The Electronic Journal of Combinatorics

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A q-Analogue of de Finetti's Theorem

Alexander Gnedin

Grigori Olshanski

PDF

Origins:

- quasi-invariant measures on 0/1 sequences
- $GL(\infty, q)$ - invariant measures on subspaces

Definition: Fix $q>0$. We say that a random sequence $(\xi_i)_{i\geq 1} \in \{0,1\}^{\infty}$ is *q-exchangeable* if

$$\mathrm{Prob}(\xi_1=x_1,\dots,\xi_{i-1}=x_{i-1},\underbrace{\xi_i=x_{i+1},\xi_{i+1}=x_i}_{\text{swapped}},\xi_{i+2}=x_{i+2},\dots,\xi_n=x_n)$$

$$= q^{x_i-x_{i+1}}\,\mathrm{Prob}(\xi_1=x_1,\dots,\xi_n=x_n)$$

[For all $1\leq i<n$ and all $x_1,\dots,x_n \in \{0,1\}$]

Meaning: At $q=1$, the measure is $S(\infty)$-invariant.
At general $q>0$, it is quasi-invariant: changes by multiplication with a controlled constant

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Example: Suppose $\mathrm{Prob}(1,0,0)=c$, then
$\mathrm{Prob}(0,1,0)=c\cdot q$ and $\mathrm{Prob}(0,0,1)=c\cdot q^2$.
Moving 1 to the right multiplies the probability by q.

Exercise 1: As usual, we can identify $0/1$ sequences with paths in the Pascal graph. At $q=1$, invariant measure corresponds to a central measure on paths [All paths with the same end-point have the same probability]
At general $q>0$ the notion of centrality changes: now given the end-point, the probability of a path is proportional to the product of weights over its edges. Find these weights.

Observation: If we swap $(z_1, z_2, \ldots) \leftrightarrow (1-z_1, 1-z_2, \ldots)$ and simultaneously invert $q \leftrightarrow q^{-1}$, the notion of q-exchangeability remains the same.

Hence, we can assume without loss of generality that

$$0 < q < 1$$

Theorem 1: Assume $0<q<1$. Extreme q-exchangeable probability measures on $0/1$ sequences are parameterized by points of $\{0,1,2,\dots\}\cup\{\infty\}$. They are:

- ∞ - measure assigns probability 1 to $(1,1,1,\dots)$
- 0 - measure assigns probability 1 to $(0,0,0,\dots)$
- K - measure is supported on sequences with K ones at positions $0<y_1<y_2<\dots<y_K$

Denoting also $y_0=0$, the spacings $y_{i+1}-y_i-1$ are independent and geometrically distributed:

$$\mathrm{Prob}\left(y_{i+1}-y_i-1=m\right)=\left(1-q^{K-i}\right)\cdot q^{m\cdot(K-i)}, \quad m=0,1,2,\dots \quad 0\le i\le K-1$$

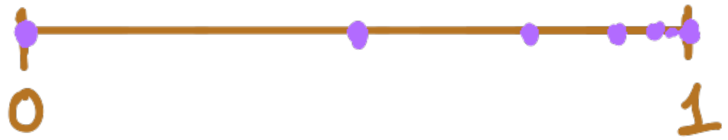
Sharp contrast with $q=1$:

- At $q=1$ all but two measures have infinitely many 0s and 1s
- At $q\neq 1$ all but one measure have finitely many 1s

.... However?

Theorem 2: Identify measures of Theorem 1 with points

$\{1\} \cup \{1-q^k\}_{k\geq 0} \subset [0,1]$



Then as $q \to 1$ the points fill entire segment $[0,1]$ and the measures converge to i.i.d. Bernoulli measures of parameter $p \in [0,1]$ $\quad (\mathrm{Prob}(\eta_i = 1) = p)$

Proof of Theorem 2: Assume that $q \to 1$ and $k \to \infty$ in such a way that $q^k \to 1-p$. We need to show that the measures of Theorem 1 converge to i.i.d. p-Bernoulli.

Indeed, the spacing between 1's becomes $\mathrm{Geom}(q^k) \to \mathrm{Geom}(1-p)$

But $\mathrm{Geom}(1-p)$ is precisely the spacing between 1's in i.i.d. p-Bernoulli: $\mathrm{Prob}(\underbrace{0 \ldots 0}_{\text{spacing } m} 1) = p \cdot (1-p)^m$, $\quad m \geq 0$.

Sketch of the proof of Theorem 1: The supports of the given measures are disjoint, hence, it suffices to show that they form the Martin boundary.

Thus, we need to choose $0 \le K(n) \le n$ for $n=1,2,\ldots$, consider q-exchangeable measure on all sequences of length n with precisely $K(n)$ 1's and $(n-K(n))$ 0's.

We need to find limits of such measures as $n \to \infty$

- $K(n)=0$. Then the only sequence is $(0,\ldots 0)$ and we converge to 0-measure of Theorem 1 as $n \to \infty$

- $K(n)=1$. Then we deal with measure with weights

$$\mathrm{Prob}(\underbrace{0\ldots0}_{m}1\underbrace{0\ldots0}_{n-m-1}) = \frac{1-q}{1-q^n}\cdot q^m, \quad 0 \le m \le n-1.$$

$$= \frac{1}{1+q+\ldots+q^{n-1}}$$

As $n \to \infty$ we converge to 1-measure of Theorem 1

- More generally, suppose that $K(n) \to K$ as $n \to \infty$. Then for large n we deal with measure on $0/1$ sequences with 1's at positions $0 < y_1 < \ldots < y_K \le n$ and weight $\mathrm{Prob}(y_1, \ldots y_K) = \frac{1}{Z} \, q^{y_1 + \ldots + y_K}$ $\quad (*)$

[Indeed, q-exchangeability of such measure is clear]

Here Z is a normalization constant, making totall mass $=1$. [In fact, $Z = q^{(\ldots)} \cdot$ q-Binomial coefficient. See q-Binomial theorem at https://en.wikipedia.org/wiki/Gaussian_binomial_coefficient]

$$q^{y_1 + \ldots + y_K} = q^{K \cdot y_1} \cdot q^{(K-1)(y_2 - y_1)} \cdot \ldots \cdot q^{(y_K - y_{K-1})} \qquad (**)$$

This gives K-measure of Theorem 1 as $n \to \infty$

- The final case is $K(n) \to \infty$ as $n \to \infty$. We would like to show that in the limit we get ∞-measure of Theorem 1.

Omitting some details, this can be seen either from (*): if, say, $y_1>1$, then $y_2>2$, $y_3>3$, ... and (*) gets multiplied at least by $q^{k(n)}$ [which is very small] compared to the minimal configuration $y_1=1$, $y_2=2$, ... Or from (**): $q^{k(n)\cdot y_1}$ factor makes $y_1=1$, $q^{(k(n)-1)(y_2-y_1)}$ factor makes $y_2-y_1=1$ as $n\to\infty$, etc. $\blacksquare$

Exercise 2: Check (directly from definition) that the measures of Theorem 1 are, indeed, q-exchangeable

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Remark 2: For $q>1$, the answer in an analogue of Theorem 1 is the same, but with roles of 0's and 1s interchanged and q replaced by q^{-1}

There is also an analogue of Theorem 2 for $q>1$.

Interpretation; infinite-dimensional Grassmanian over a finite field.

$\mathbb{F}_q$ – finite field with q elements

V_n – n-dimensional vector space : $(x_1, \dots x_n)$ $[x_i \in \mathbb{F}_q]$

$V_\infty = \bigcup_{n=1}^{\infty} V_n$ – sequences $(x_1, x_2, \dots)$, such that $x_i = 0$ for large enough i.

[Such infinite-dimensional spaces with countable basis are rarely studied over $\mathbb{R}$, but there is no L_p over $\mathbb{F}_q$ and we have no choice]

$Gr(V_\infty)$ – all (linear) subspaces in V_∞

Grassmanian

$GL(n, F_q)$ – all invertible $n \times n$ matrices with elements from $\mathbb{F}_q$

$GL(\infty, F_q) = \bigcup_{n=1}^{\infty} GL(n, F_q)$ – infinite matrices of the form $\begin{pmatrix} \text{invertible} & 0 \\ 0 & \begin{matrix} 1 & \\ & \ddots \end{matrix} \end{pmatrix}$

$GL(n, F_q)$ acts in V_n. $GL(\infty, \mathbb{F}_q)$ acts in V_∞ and in $Gr(V_\infty)$

Random $GL(\infty, F_q)$-invariant subspace $X \in Gr(V_\infty)$?

Theorem 3: $GL(\infty, \mathbb{F}_q)$-invariant probability measures on $Gr(V_\infty)$ are in bijection with q-exchangeable laws of sequences of $0/1$. The correspondence is given by projection $\pi : Gr(V_\infty) \to \{0,1\}^\infty$

$$\pi(X) = \left(\dim(X \cap V_1), \dim(X\cap V_2) - \dim(X \cap V_1), \ldots, \dim(X \cap V_n) - \dim(X \cap V_{n-1}), \ldots\right)$$

dimension of space

We do not give a proof — see Section 5 of Gnedin-Olshanski.
Idea: $X \leftrightarrow$ all $X \cap V_n$. $GL(n, F_q)$-orbit of $X \cap V_n$ is fixed by its dimension

Corollary: Ergodic random $GL(\infty, \mathbb{F}_q)$-invariant subspaces X are:

- $X = \{0\}$ – subspace consisting of one element $(0, 0, \ldots)$
- Random subspace of finite codimension $k = 0, 1, 2, \ldots$. In particular, for $k=0$, we have $X = V_\infty$.

Additional literature:

More general sequences than just 0/1

q-exchangeability via quasi-invariance

Alexander Gnedin, Grigori Olshanski

Ann. Probab. 38(6): 2103-2135 (November 2010). DOI: 10.1214/10-AOP536

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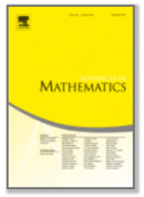
Volume 254, 20 March 2014, Pages 331-395



Asymptotic representation theory of $GL(n, \mathbb{F}_q)$

Finite traces and representations of the group of infinite matrices over a finite field

Vadim Gorin [a, b], Sergei Kerov [1], Anatoly Vershik [c, d]

and more complicated measures on subspaces over $\mathbb{F}_q$

Random matrices over $\mathbb{F}_q$

BULLETIN (NEW SERIES) OF THE AMERICAN MATHEMATICAL SOCIETY

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Random matrix theory over finite fields

Author: Jason Fulman

Journal: Bull. Amer. Math. Soc. **39** (2002), 51-85

[Submitted on 3 Feb 2021]

Infinite-dimensional groups over finite fields and Hall-Littlewood symmetric functions

Cesar Cuenca, Grigori Olshanski

Recall that at $q=1$: Pascal graph $\hookrightarrow$ Gelfand-Tsetlin graph.

This embedding lifts to the q-level

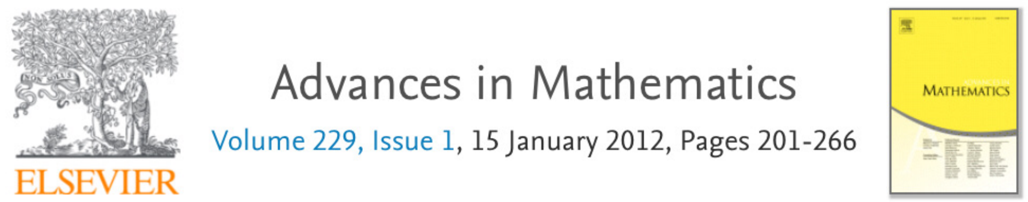
Definition: q-Gelfand-Tsetlin graph q-GT has levels formed by signatures $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N$ and cotransition probabilities

$$L_{N+1\to N}(\lambda \to \mu) = \begin{cases} (1-q)\cdot\ldots\cdot(1-q^N)\cdot q^{\sum_{i=1}^{N}(\mu_i+N-i)} \cdot \dfrac{\prod_{i<j}\left(q^{\mu_j+N-j}-q^{\mu_i+N-i}\right)}{\prod_{i<j}\left(q^{\lambda_j+N+1-j}-q^{\lambda_i+N+1-i}\right)}, & \text{if } \mu \prec \lambda \\ 0, & \text{otherwise.} \end{cases}$$

interlace, i.e. $\lambda_1 \geq \mu_1 \geq \lambda_2 \geq \ldots \geq \mu_N \geq \lambda_{N+1}$

- q-Pascal is embedded a subgraph of signatures $(1\geq 1 \geq \ldots \geq 1 \geq 0 \ldots \geq 0)$

  [Exercise 1 on Slide 4 leads to that]
- At $q=1$ back to GT of Week 8


Advances in Mathematics
Volume 229, Issue 1, 15 January 2012, Pages 201-266

The q-Gelfand–Tsetlin graph, Gibbs measures and q-Toeplitz matrices

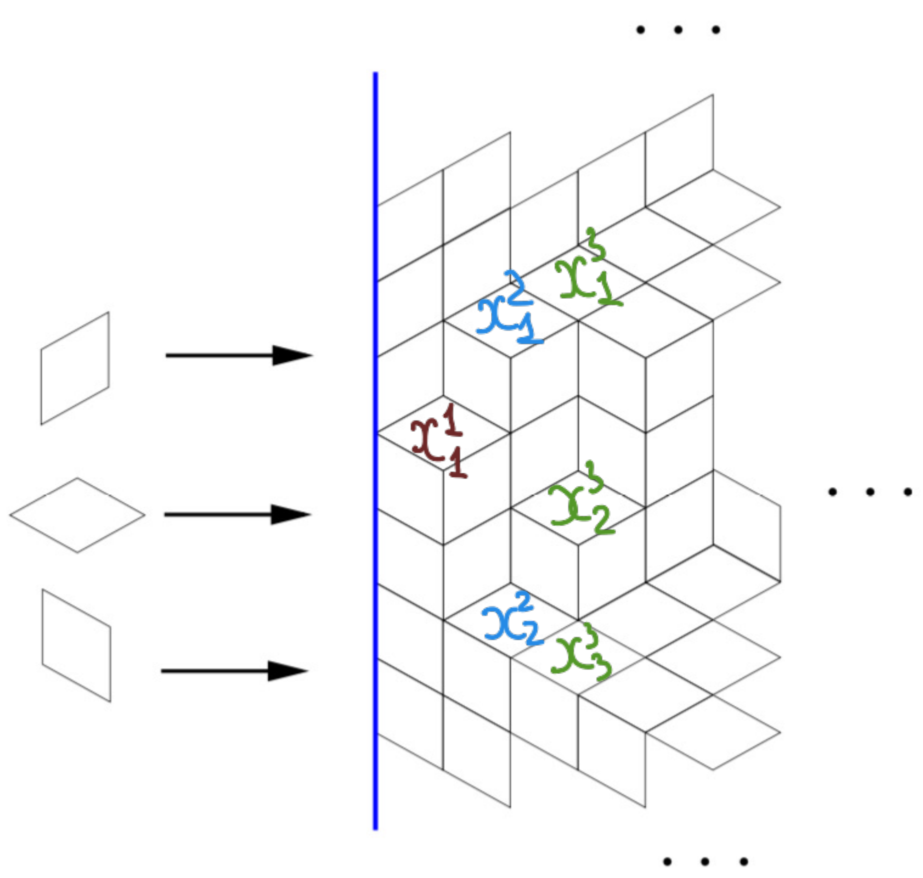
Vadim Gorin

- This is also reminiscent of Week 10, Slide 10, Proposition 2
- The normalization $\sum_{\mu} L_{N+1\to N}(\lambda \to \mu) = 1$ can be checked using Schur polynomials

Motivations for q-GT:

see Week 8, Slide 10

1. Combinatorial/probabilistic.



Recall that paths in GT (or q-GT) are in bijection with lozenge tilings, which can be also viewed as two-dimensional stepped surfaces in 3d-space.

Central measures in GT

$\Updownarrow$

Conditionally uniform random tilings

Multiplying links $L_{N+1\to N} \cdot L_{N\to N-1} \cdot \ldots \cdot L_{2\to 1}$, we get

Central measures in q-GT $\Updownarrow$

Tilings with weight $q^{\sum_{k=1}^{N}\sum_{i=1}^{k} x_i^k} \sim q^{\text{Volume under the surface}}$

This is the simplest possible non-uniform weight on tilings

2. Representation theoretic. Recall that for GT $\mathrm{Dim}_N(\lambda)$ = dimension of irreducible representation of $U(N)$ indexed by signature ("highest weight") $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$

Similarly, for q-GT, setting $\mathrm{Dim}_N^q(\lambda) = \sum_{\text{paths } \emptyset \to \lambda} q^{\text{Volume}}$

we have $\mathrm{Dim}_N^q(\lambda)$ = quantum dimension of representation of quantized universal envelopping algebra ("quantum group") $\mathcal{U}_q(gl_N)$ indexed by signature $\lambda_1 \geq \dots \geq \lambda_N$

Such representation has the same dimension as vector space, as the $U(N)$ representation. However, there is a distinguished element q^ρ in quantum group and by many reasons it is more natural to consider its trace in representation (= quantum dimension).

Similarly, there are quantum traces: $\mathrm{Trace}(T(A q^\rho))$

usual trace representation element of quantum group

__Theorem 4:__ Assume $0<q<1$. Extreme coherent systems in q-GT are parameterized by infinite sequences of integers: $\nu_1 \le \nu_2 \le \nu_3 \le \dots \in \mathbb{Z}$

Let $\lambda^{(1)} \prec \lambda^{(2)} \prec \lambda^{(3)} \prec \dots$ be a path distributed according to (q-) central measure corresponding to such sequence. Then

$$\lim_{N\to\infty} \lambda^{(N)}_{N+1-i} = \nu_i \qquad [\text{No normalizations!}]$$

Suppose that $\nu_1 \ge 0$. Then $\lambda^{(1)}_1 \ge 0$ almost surely and

$$\sum_{m=0}^{\infty} \mathrm{Prob}(\lambda^{(1)}_1 = m)\, \underbrace{(1-x)(1-qx)\dots(1-q^{m-1}x)}_{q\text{-interpolation polynomial}} = \prod_{i\ge 1} \frac{1-q^{i-1}x}{1-q^{\nu_i+i-1}x}$$

- The structure of q-GT is invariant under simultaneous shifts of all coordinates $\lambda^k_i \mapsto \lambda^k_i + c$. Same shifts apply to $\nu_1 \le \nu_2 \le \dots$
- The law of $(\lambda^k_1, \dots, \lambda^k_k)$ can be similarly described in terms of q-Interpolation Schur polynomials

Four proofs of Theorem 4 in the literature

The Annals of Probability
2015, Vol. 43, No. 6, 3052–3132
DOI: 10.1214/14-AOP955

ASYMPTOTICS OF SYMMETRIC POLYNOMIALS WITH APPLICATIONS TO STATISTICAL MECHANICS AND REPRESENTATION THEORY

By Vadim Gorin[1] and Greta Panova[2]

ELSEVIER
Journal of Functional Analysis
Volume 270, Issue 1, 1 January 2016, Pages 375-418

A quantization of the harmonic analysis on the infinite-dimensional unitary group

Vadim Gorin [a, b], Grigori Olshanski [b, c]

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Advances in Mathematics
Volume 229, Issue 1, 15 January 2012, Pages 201-266

The q-Gelfand–Tsetlin graph, Gibbs measures and q-Toeplitz matrices

Vadim Gorin

Moscow Mathematical Journal
Volume 14, Issue 1, January–March 2014 pp. 121–160.

The Boundary of the Gelfand–Tsetlin Graph: New Proof of Borodin–Olshanski's Formula, and its q-Analogue

Authors: Leonid Petrov

We will only sketch some ideas.

As usual, we compute the Martin boundary first and start from $\lambda_1 \geq \dots \geq \lambda_N$. We act on it wih $(q\text{-})$ links to get a (random) signature $\mu_1 \geq \dots \geq \mu_K$. We aim to send $N \to \infty$ keeping K fixed. Note that due to interlacing, $\mu_K \geq \lambda_N$, $\mu_{K-1} \geq \lambda_{N-1}$, ..., $\mu_1 \geq \lambda_{N-K+1}$ almost surely.

This was also true at $q=1$ and in β-spectra graph. However, the new feature of $0<q<1$ setting is that

$$\mathrm{Prob}(\mu_K = \lambda_N;\ \mu_{K-1} = \lambda_{N-1};\ \dots;\ \mu_1 = \lambda_{N-K+1}) > c > 0$$

with c staying away from 0 as $N \to \infty$.

This can be proven purely probabilistically by analyzing the cotransition probabilities, and it readily implies the necessity of $\lambda(N)_{N-i+1} \to \nu_i$, $i=1,2,\dots$

[as otherwise a part of the measure is escaping to $\pm\infty$]

However, this appoach does not give any formulas for the limiting coherent systems (Martin boundary)

The latter can be obtained by using Schur functions [See week 6, slide 5]

Proposition 1: For any $\lambda = (\lambda_1 \geq \dots \geq \lambda_{N+1})$ in q-GT we have

$$\frac{S_\lambda(x_1,\dots,x_N,q^{-N})}{S_\lambda(1,q^{-1},\dots,q^{-N})} = \sum_{\mu=(\mu_1\geq\dots\geq\mu_N)} L_{(N+1)\to N}(\lambda\to\mu)\,\frac{S_\mu(x_1,\dots x_N)}{S_\mu(1,q^{-1},\dots q^{1-N})}$$

Thus, cotransition probabilities are encoded through branching rules for Schur polynomials upon restrictions to smaller # of variables.

We need auxiliary lemmas:

Lemma 1: $$S_\lambda(1, q, \dots q^{N-1}) = \prod_{i<j} \frac{q^{\lambda_i+N-i} - q^{\lambda_j+N-j}}{q^{N-i} - q^{N-j}}$$

Proof. We use Vandermonde determinant: $\det[x_i^{N-j}]_{i,j=1}^{N} = \prod_{i<j}(x_i - x_j)$

Then $$S_\lambda(1, \dots, q^{N-1}) = \frac{\det[q^{(N-j)(\lambda_i+N-i)}]}{\prod_{i<j}(q^{N-i} - q^{N-j})} = \prod_{i<j} \frac{q^{\lambda_i+N-i} - q^{\lambda_j+N-j}}{q^{N-i} - q^{N-j}} \qquad \blacksquare$$

Lemma 2. $$S_\lambda(ax_1, \dots, ax_N) = a^{\lambda_1+\dots+\lambda_N} S_\lambda(x_1, \dots x_N)$$

[Immediate from definition]

Lemma 3. $$S_\lambda(x_1, \dots, x_N, t) = \sum_{\mu \prec \lambda} t^{\Sigma\lambda_i - \Sigma\mu_i} S_\mu(x_1, \dots x_N)$$

interlace

For the proof of Lemma 3, repeat the argument of Week 11, Slides 16-17 or see MATH 740, Lecture 4, Slides 3-5

Proof of Proposition 1: The last lemma implies

$$\frac{S_\lambda(x_1,\dots,x_N,q^{-N})}{S_\lambda(1,q^{-1},\dots,q^{-N})} = \sum_{\mu\prec\lambda} q^{-N\left(\sum_{i=1}^{N+1}\lambda_i - \sum_{i=1}^{N}\mu_i\right)} \cdot \frac{S_\mu(1,\dots q^{1-N})}{S_\lambda(1,\dots q^{-N})} \cdot \frac{S_\mu(x_1,\dots x_N)}{S_\mu(1,\dots q^{1-N})}$$

It remains to identify $L_{(N+1)\to N}(\lambda\to\mu) = q^{-N\left(\sum_{i=1}^{N+1}\lambda_i - \sum_{i=1}^{N}\mu_i\right)} \cdot \frac{S_\mu(1,\dots q^{1-N})}{S_\lambda(1,\dots q^{-N})}$ using Lemmas 1 and 2. $\blacksquare$

Corollary 1: $\sum_{\mu} L_{(N+1)\to N}(\lambda\to\mu) = 1.$

Proof: Set $(x_1,\dots,x_N)=(1,q^{-1},\dots q^{1-N})$ in Proposition $\blacksquare$

Corollary 2: Fix $\lambda=(\lambda_1,\dots\lambda_N)$ and let $\lambda^{(1)}\prec\lambda^{(2)}\prec\dots\prec\lambda^{(N)}=\lambda$ be a q^{vol}-weigted random path. [Trajectory of chain $L_{N\to N-1}$; $L_{N-1\to N-2}$; ..]

Then $$\sum_{m\in\mathbb{Z}} \mathrm{Prob}(\lambda^{(1)}=m)\, x^m = \frac{S_\lambda(x,q^{-1},\dots q^{1-N})}{S_\lambda(1,q^{-1},\dots,q^{1-N})}$$

Proof. We iterate Proposition 1 $(N-1)$ times. ▯

Remark 2: Setting $q=1$ in Corollary 2 we get Week 8, Slide 8, Th. 4

Remark 3: There is an immediate analogue of Corollary 2 for the generating function of distribution of $(\lambda_1^{(k)}, \dots \lambda_k^{(k)})$
[Write it down!]

Conclusion: In order to find the Martin boundary of q-GT, we need to find limits of $\frac{S_\lambda(x, q^{-1}, \dots q^{1-N})}{S_\lambda(1, \dots q^{1-N})}$ as $N \to \infty$ and show that the last coordinates of λ should stabilize to $\nu_1 \le \nu_2 \le \dots$

Proposition 2:

$$\frac{S_\lambda(x, q^{-1}, \dots, q^{1-N})}{S_\lambda(1, \dots, q^{1-N})} = \left[\prod_{i=1}^{N-1} \frac{1-q^i}{1-x q^i}\right] \cdot \frac{\ln q}{2\pi\sqrt{-1}} \cdot \oint_{\{\lambda_i+N-i\}} \frac{x^z \, dz}{\prod_{i=1}^{N} \left(1 - q^{-z} q^{\lambda_i+N-i}\right)}$$

where the integration goes over a positively ↺ oriented contour enclosing the poles at $\lambda_i + N - i$, $i = 1, 2, \dots N$.

Proof of Proposition 2: $S_\lambda(x_1,\dots x_N)\cdot\prod_{i<j}(x_i-x_j)=\det[x_i^{\lambda_j+N-j}]$

$$=\sum_{k=1}^{N}(-1)^{k-1}x_1^{\lambda_k+N-k}\cdot\det[x_i^{\lambda_j+N-j}]_{i\geq 2,\ j\neq k}$$

We can plug $x_i=q^{1-i}$, $i=2,3,\dots,N$, and $x_1=x$.

Determinant is again Vandermonde, as in Lemma 1, and is, therefore, explicitly evaluated. Hence, $\frac{S_\lambda(x,\dots q^{1-N})}{S_\lambda(1,\dots q^{1-N})}$ is an explicit sum of N terms.

These terms match the expansion of the contour integral as a sum of N residues at poles $z=\lambda_j+N-j$. ∎

Remark 4: Sending $q\to 1$, we get formulas of Week 8, Slide 9

Sending $N\to\infty$ in the result of Proposition 2 is not hard:

The integrand converges towards $\frac{x^z}{\prod_{i=1}^{\infty}(1-q^{-z}q^{\lambda_i+i-1})}$, and the prefactor becomes $\prod_{i=1}^{\infty}\frac{1-q^i}{1-xq^i}$. [The contour needs a deformation to make sense as $N\to\infty$]

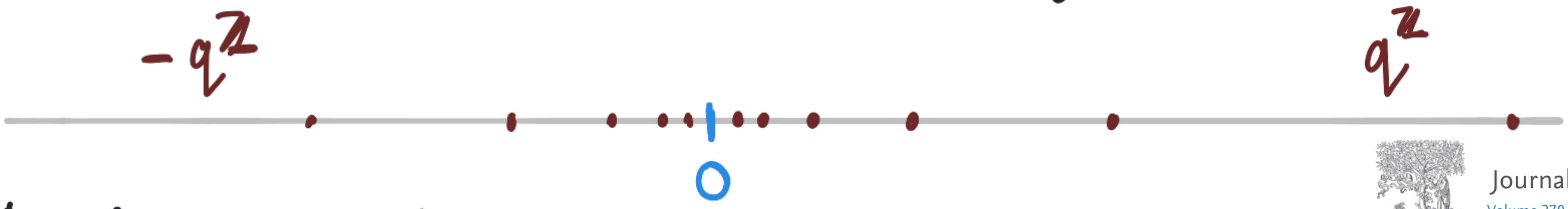
Resummations eventually lead to the result of Theorem 4.

We do not provide further details on Theorem 4, but let us mention that it has a remarkable generalization.

If we interpret $(\lambda_1, \dots, \lambda_N)$ as N-particle configuration $\{q^{\lambda_i+N-i}\}_{i=1}^N$ on the lattice $q^{\mathbb{Z}}$, then the link of Slide 13 takes the form $L_{(N+1)\to N}(X \to Y) =$

$$= (1-q)_{\infty} \, (1-q^N) \prod_{i=1}^{N} |y_i| \cdot \frac{\prod_{i<j} (y_i - y_j)}{\prod_{i<j} (x_i - x_j)} \qquad [\,Y \text{ interlaces with } X\,]$$

One can extend the same formula to configurations on two-sided lattice.



- Analogue of Theorem 4
- Important coherent systems, which do not exist on q-GT
- Representation-theoretic meaning of doubling is unknown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A quantization of the harmonic analysis on the infinite-dimensional unitary group

Vadim Gorin [a, b], Grigori Olshanski [b, c]