University of Wisconsin – Madison

Math 833. Topics in probability: Branching graphs and integrable probability, Spring 2021

Vadim Gorin

Credits: 3
Course attributes: Counts toward 50% graduate coursework requirement
Website: `https://canvas.wisc.edu/courses/235500`
Meeting time and location: ONLINE.
Instructional mode: Lecture notes and short videos on the material are uploaded weekly on canvas. Instructor expects to interact with students during the live office hours, over the Piazza forum, and through discussions of weekly exercises accompanying lectures.

Welcome and introductory meeting is on Friday, Jan 29, 12.30pm ("Zoom" section on Canvas.)

Instructor: Vadim Gorin (vadicgor@math.wisc.edu) Questions can be asked and answered in Zoom on Fridays between 1pm and 2pm (see "Zoom" section of the Canvas homepage of the class). In addition, please, use Piazza to ask more questions. Finally, additional one-on-one Q&A online sessions with V.G. can be arranged through e-mail requests.

Course description: What are all sequences of random variables which are invariant under permutations? What are all possible Appell sequences of real-rooted polynomials? What are all conjugation-invariant laws of infinite random Hermitian matrices? We will answer these and many other questions under a unifying umbrella of classification of boundaries of branching graphs and discover that the answers lead to numerous exactly-solvable stochastic systems of integrable probability.

Prerequisites: Graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International). Prior exposure to MATH 733 (Theory of Probability) and MATH 740 (Symmetric functions) can be helpful, but is not required.

Learning outcomes. Students will master the modern theory of branching graphs, giving rise to numerous important distributions of integrable probability.

Grading. The final grade is based on 12 weekly exercises (24%), one polished exercise (16%), mid-semester homework (30%), and final oral discussion/examination (30%).

The grades are not curved.

Labs and discussion sessions: none.

Weekly schedule. At the beginning of each week a new portion of lecture notes and videos is uploaded on canvas. The notes will include a couple of exercises. Students have one week to submit solutions to the exercises. In general, solutions do not have to be detailed, but you need to demonstrate that you understand the material. The grading of this part will be relaxed (0 points = no solution submitted, 1 point = solution of some, but not all exercises submitted, 2 points = full submission received), yet I still encourage you to invest efforts into weekly assignments, as this is crucial for proper understanding of the material.

Simultaneously, each week we are going to have 1-2 people responsible for careful and detailed typing in LaTex of solutions of the exercises. I will give feedback to these people and they are expected to polish their solutions to perfection. The final version will be posted on canvas. Please, sign up at `https://bit.ly/3ivjb2o` to choose your week. This part contributes 16 points to your grade.

Mid-semester homework. In addition, we will have a homework, which will be posted March 1 and due April 1. You should expect the problems in this homework to be harder than the weekly exercises. This part contributes 30 points to your grade.

Collaboration. For all assignments you are encouraged to collaborate and use any sources except for those where the solutions to exactly the same problems are written. However, all your collaborators and used outside sources should be clearly indicated on your homework when you hand it in (otherwise, it will be treated as a plagiarism). Each person should write down the solutions individually.

Oral discussion/examination. In early April I will split all students into groups of 2-3 people and each group will be given a list of topics to prepare. We will meet with each group on Zoom during the last week of classes and I will be asking the students from the group to explain the topic ("A, please, tell me the statement of Theorem X. Thanks. B, could you now tell me the key idea of the proof? Thanks. C, how is this theorem used in random matrix theory?"). During the Zoom meeting you are expected to answer my questions right away without additional preparation time (and without using any materials).

Piazza. An online forum is accessible through the class canvas page. Please, ask and answer questions their. The instructor will also be answering questions there regularly (if something stays unanswered for several days, please, follow up by e-mail). You are allowed to either sign with your name or to post anonymously.

Academic Integrity. By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madisons community of scholars in which everyones academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to `studentconduct.wiscweb.wisc.edu/academic-integrity/`.

Accommodations for students with disabilities. The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [V.G.] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [V.G.], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA.

Diversity and inclusion. Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background people who as students, faculty, and staff serve Wisconsin and the world. `https://diversity.wisc.edu/`

Preliminary list topics for the class.

- Introduction to integrable probability and classification problems. What are the stochastic systems, which are exceptionally nice? How to find them?
- Real-rooted polynomials and their derivatives.
- De Finetti theorem and sequences of i.i.d Bernoully random variables.
- General theory of boundaries of branching graphs. Choquet simplices.
- Young graph and its boundary. Characters of $S(\infty)$, Plancherel measure on Young diagrams.
- Gelfand-Tsetlin graph and its boundary. Connection to random lozenge tilings and to Schur polynomials.
- Graph of spectra as a degeneration of the Gelfand-Tsetlin graph. Connection to Wigner and Wishart random matrices.
- General β random matrix theory and a loop to real-rooted polynomials.
- q–deformations of boundaries.
- Harmonic analysis on branching graphs.