University of Wisconsin – Madison

Math 740. Symmetric functions, Fall 2020

Vadim Gorin

Credits: 3
Course attributes: Counts toward 50% graduate coursework requirement
Website: `https://canvas.wisc.edu/courses/219345`
Meeting time and location: Tue, Th 11:00 am-12:15 PM.
ONLINE.
Instructional mode: Lecture notes and short videos on the material of each lecture are uploaded on canvas. During the lecture time the instructor is online in Zoom room and ready to answer the questions about the lecture material (or any other questions!).
Welcome and introductory meeting is on Thursday, Sep. 3 at 11am.

Instructor: Vadim Gorin (vadicgor@math.wisc.edu) Questions can be asked and answered in Zoom room during the lecture time (see "Office hours" section of the Canvas homepage of the class). In addition, please, use Piazza to ask more questions. Finally, additional one-on-one Q&A online sessions with V.G. can be arranged through e-mail requests.

Course description: We will study various classes of symmetric polynomials and relations between them. The objects range from basic (elementary symmetric functions, complete homogeneous symmetric functions, power sums) to advanced (Schur, Jack, Hall-Littlewood, and Macdonald symmetric functions). I will also outline connections and applications of symmetric polynomials to enumerative combinatorics, representation theory, and integrable probability.

Prerequisites: Graduate/professional standing or member of the Pre-Masters Mathematics (Visiting International). MATH 741 (algebra) is recommended.

Learning outcomes. Students will master the modern theory of symmetric functions and learn its connections to various branches of mathematics, such as combinatorics, representation theory, and integrable probability.

Grading. The final grade is based on regular homeworks (25%), Midterm (40%) and final assignment (35%). For the homeworks, we will have five of them.
The grades are not curved.

Labs and discussion sessions: none.

Suggested literature.

1. R. P. Stanley, Enumerative Combinatorics (volume 2)
2. I. G. Macdonald, Symmetric Functions and Hall Polynomials (second edition)

Midterm and final homework policy. No collaboration or outside assistance is allowed for the midterm exam (on Thursday, Oct 22, during the class time) and final take-home assignment (due on Thursday, Dec 10). You are always allowed to use the course materials (notes, videos) and suggested textbooks.

Homework policy. For regular problem sets you are allowed to collaborate and use any sources except for those where the solutions to exactly the same problems are written. However, all your collaborators and used outside sources should be clearly indicated on your homework when you hand it in (otherwise, it will be treated as a plagiarism). Each person should write down the solutions individually.

Neatness and clarity are essential. You have to write enough to show that you understand the flow of ideas and that you are not jumping to unjustified conclusions. You are encouraged to use LaTeX to typeset your solutions, however, this is not required.

At the date when the homeworks are due, it should be submitted online in canvas before noon. No late submissions will be accepted.

Homeworks are due on Fridays at noon: Sep 18, Oct 2, Oct 16, Nov 6, Nov 20.

Piazza. An online forum is accessible through the class canvas page. Please, ask and answer questions their. The instructor will also be answering questions there regularly (if something stays unanswered for several days, please, follow up by e-mail). You are allowed to either sign with your name or to post anonymously.

Academic Integrity. By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madisons community of scholars in which everyones academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to `studentconduct.wiscweb.wisc.edu/academic-integrity/`.

Accommodations for students with disabilities. The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [V.G.] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [V.G.], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA.

Diversity and inclusion. Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background people who as students, faculty, and staff serve Wisconsin and the world. `https://diversity.wisc.edu/`

Key topics for the class. Motivations to study symmetric functions from other areas of mathematics. Ring of symmetric functions in finitely many and infinitely many variables. Power sums, elementary symmetric functions, complete homogeneous functions. Young diagrams enumerating bases. Schur functions and skew Schur functions: orthogonality, Cauchy identities, Weyl formulas, Jacobi-Trudy formula, combinatorial formula, difference/differential operators, links to lozenge tilings and representations of symmetric/unitary groups. Robinson-Schensted-Knuth algorithm, longest increasing subsequences. Littlewood-Richardson coefficients. Schur polynomials through vertex models and Yang-Baxter relation. Shifter Schur functions and interpolation polynomials, binomial formula. Extensions to Jack, Hall-Littlewood, and Macdonald polynomials.