ME/EMA 540

Experimental Vibrations & Dynamic System Analysis

Module #3: Ritz Analysis & Support Conditions

Ritz Analysis

- The modal parameter sensitivity equations assume a small change in the natural frequency and negligible change in mode shape.
- <u>Ritz analysis is valid for an arbitrary change, so long as the basis functions used in the analysis adequately represent the actual deformation of the structure.</u>
- Ritz approximation of displacement:

$$u(x,t) = \sum_{j=1}^{N} \psi_j(x) q_j(t)$$

- Equation of motion for the generalized coordinates.

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q\}$$

$$Q_j = \int_0^L f_x \psi_j \, dx + \sum F \psi_j(x_{Fn}, \cancel{t})$$

- where f denotes a distributed load and F denotes point loads applied at x_{Fn} (subsequent analysis on board)

Equations for M, C and K for Rod (Axial)

$$\boxed{M_{jn} = M_{nj} = \int_0^L \psi_j \psi_n \rho A \, dx + \sum m \psi_j(x_m) \psi_n(x_m)}$$

$$C_{jn} = C_{nj} = \int_0^L \gamma EA \frac{d\psi_j}{dx} \frac{d\psi_n}{dx} dx + \sum c \psi_j(x_c) \psi_n(x_c)$$

$$\boxed{K_{jn} = K_{nj} = \int_0^L EA \frac{d\psi_j}{dx} \frac{d\psi_n}{dx} dx + \sum k \psi_j(x_k) \psi_n(x_k)}$$

$$Q_j = \int_0^L f_x \psi_j \, dx + \sum F \psi_j(x_{Fn}, \cancel{t})$$

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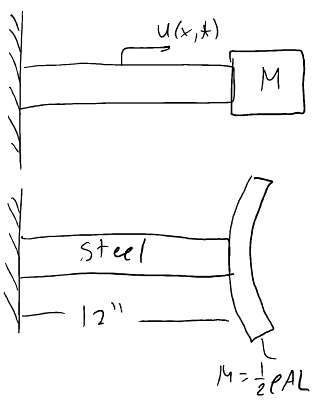
Matlab Example



$$M_{jn} = \frac{\rho A L}{(j+n+1)} + M$$

$$K_{jn} = \frac{EA}{L}\left(\frac{jn}{j+n-1}\right)$$

Let $N = 3$ $\quad M = \frac{1}{2}\rho A L$

Steel

$$M = \rho A L \begin{bmatrix} 1/3 & 1/4 & 1/5 \\ 1/4 & 1/5 & 1/6 \\ 1/5 & 1/6 & 1/7 \end{bmatrix} + \frac{1}{2}\rho A L \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$K = \frac{EA}{L} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4/3 & 3/2 \\ 1 & 3/2 & 9/5 \end{bmatrix}$$

RitzBarExample.m

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Mode Shapes

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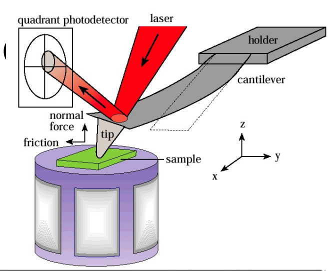
Ex: AFM Cantil



DMCMN: Experimental/Analytical Evaluation of the Effect of Tip Mass on Atomic Force Microscope Cantilever Calibration

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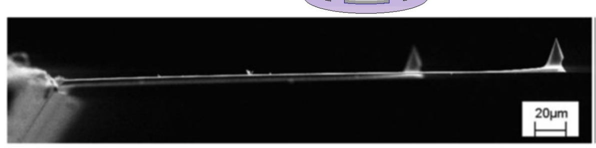

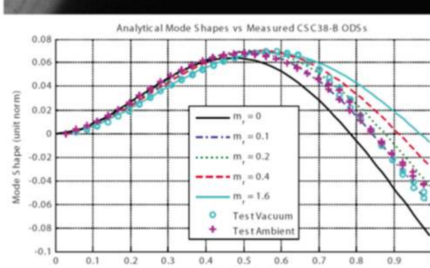

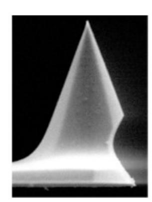

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M, C and K for Beam (Bending)

$$M_{jn} = M_{nj} = \int_0^L \psi_j \psi_n \rho A \, dx + \sum m \psi_j(x_m) \psi_n(x_m)$$

$$K_{jn} = K_{nj} = \int_0^L EI \frac{d^2\psi_j}{dx^2} \frac{d^2\psi_n}{dx^2} dx + \sum k \psi_j(x_k) \psi_n(x_k) + \sum \kappa \frac{d\psi_j}{dx}(x_\kappa) \frac{d\psi_n}{dx}(x_\kappa)$$

$$C_{jn} = C_{nj} = \int_0^L \left[\gamma EI \frac{d^2\psi_j}{dx^2} \frac{d^2\psi_{\cancel{j}\,n}}{dx^2} + c_v \psi_j \psi_n \right] dx + \sum c \psi_j(x_c) \psi_n(x_c) + \sum \chi \frac{d\psi_j}{dx}(x_\chi) \frac{d\psi_n}{dx}(x_\chi)$$

$$Q_j = \int_0^L f_z \psi_j \, dx + \sum F \psi_j(x_F) + \sum M \frac{d\psi_j}{dx}(x_M)$$

Ritz Series

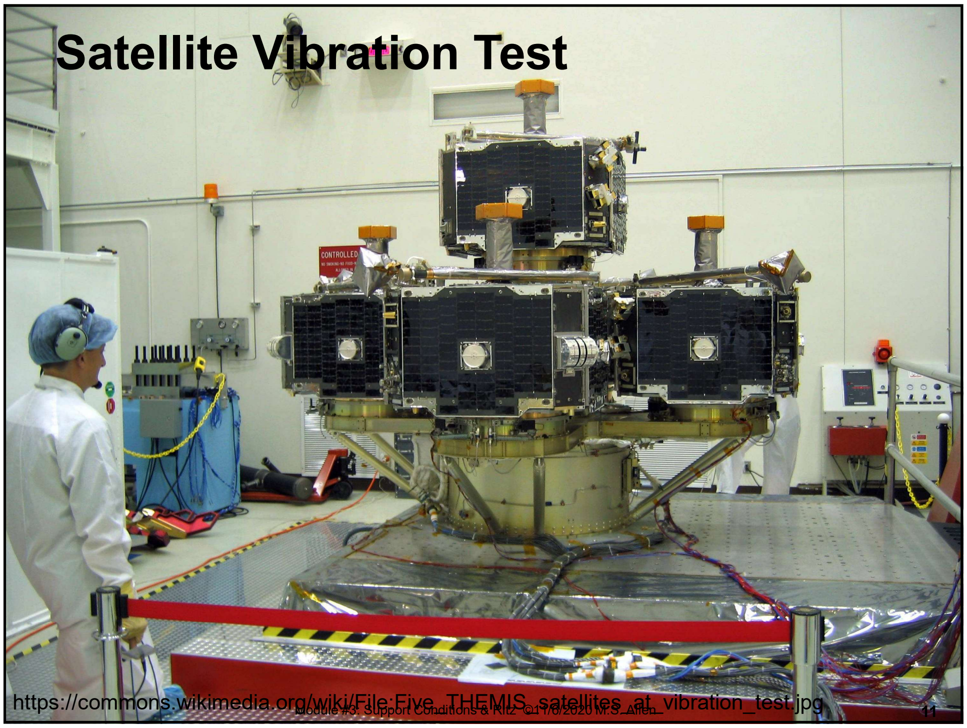
- Derivation on board
- See CantileverBeamRitz.m
  - Shows how to use fsolve to get more precise solutions to the characteristic equation.
  - Shows how to use symbolics in Matlab to find the terms to integrate in the Ritz series.

Support Conditions for a Modal Test

Support Conditions

- When testing a structure to update a finite element model, what do we do with the boundary conditions?
  - We could model them in the FEA software – but this takes extra effort. Also, how do we know where to stop?
  - We could fix the structure to a rigid foundation (fixed-interface), but these prove very difficult to build in practice.

Satellite Vibration Test



https://commons.wikimedia.org/wiki/File:Five_THEMIS_satellites_at_vibration_test.jpg

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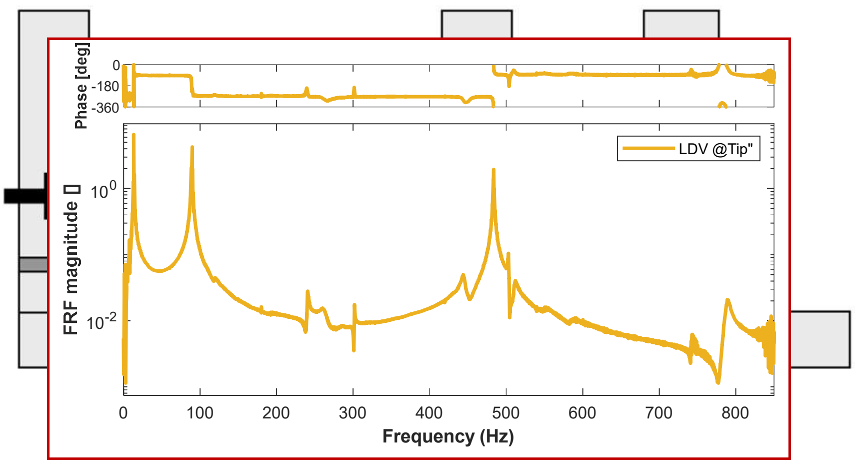
You've also seen this in HW#7



- If you look closely at the high frequency measurements you'll see additional resonances of the support structure.
- Also, the higher natural frequencies will be spaced differently than they would if the end was fixed.

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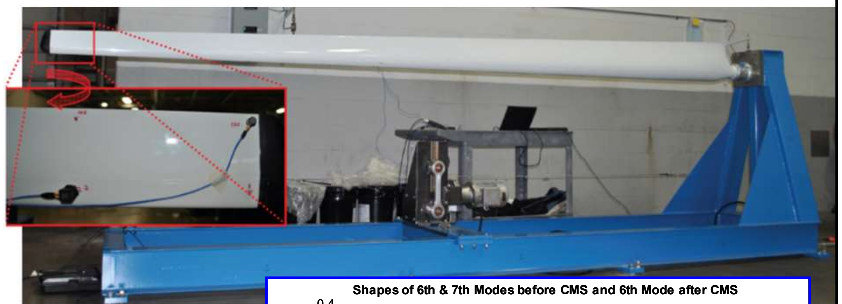
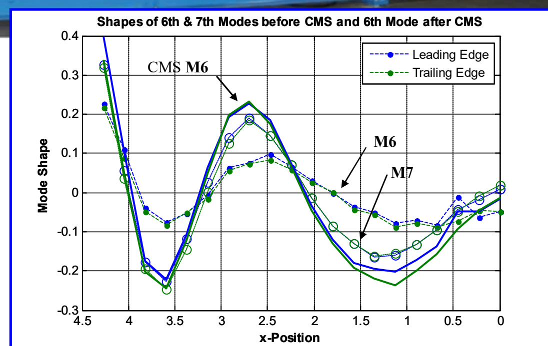
Support Conditions: Challenges





- Wind turbine blade in a rigid fatigue frame – does it behave as a fixed base?
- M6: 45.5 Hz, M5: 52.5 Hz

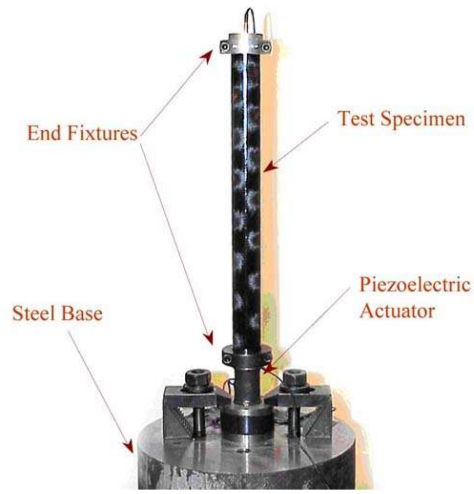
Modes After Applying Constraints

Mode Num.	Mode Shape Description	Natural Freq. (Hz)	2 SVD Constraints		3 SVD Constraints	
			f_n CMS	% Diff	f_n CMS	% Diff
1	FW B1	3.36	3.83	12.1%	3.84	12.4%
2	EW B1	5.24	5.27	0.5%	5.28	0.7%
3	FW B2	11.40	11.44	0.4%	11.64	2.1%
4	EW B2	22.42	22.52	0.4%	22.77	1.6%
5	FW B3	28.44	28.85	1.4%	29.54	3.7%
6	FW B4, Fixture+	45.50	48.92	7.0%	50.26	9.5%
7	FW B4, Fixture–	52.26	-	-	-	-
8	EW+FW	53.37	-	-	-	-
9	EW B3	58.29	56.52	-3.1%	56.96	-2.3%
10	1st Torsion	80.01	79.96	-0.1%	79.97	0.0%
11	FW B5	83.54	81.84	-2.1%	83.90	0.4%
12	EW B4	107.37	106.85	-0.5%	107.01	-0.3%
13	FW B6	118.25	115.77	-2.1%	119.75	1.2%
14	2nd Torsion	143.47	143.45	0.0%	143.54	0.0%
15	FW B7, Tors.	150.29	150.12	-0.1%	154.12	2.5%
16	FW B7, Tors.	156.21	154.18	-1.3%	-	-
17	EW B5 +FW	169.61	168.30	-0.8%	159.09	-6.6%
18	FW B7, EW B8, Torsion	184.11	183.02	-0.6%	182.97	-0.6%

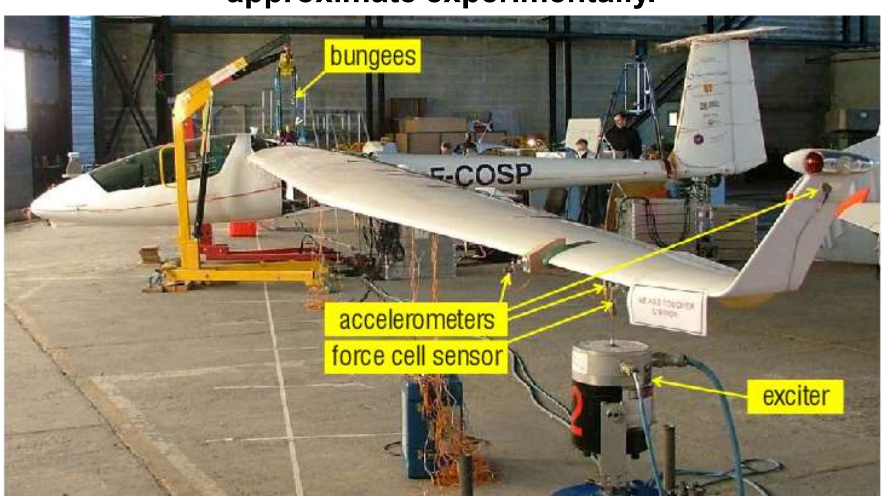
- Natural frequencies of wind turbine blade in frame.
- Modes correspond to:
  - Edgewise bending (EW)
  - Flapwise bending (FW)
  - Torsion.

Support Conditions

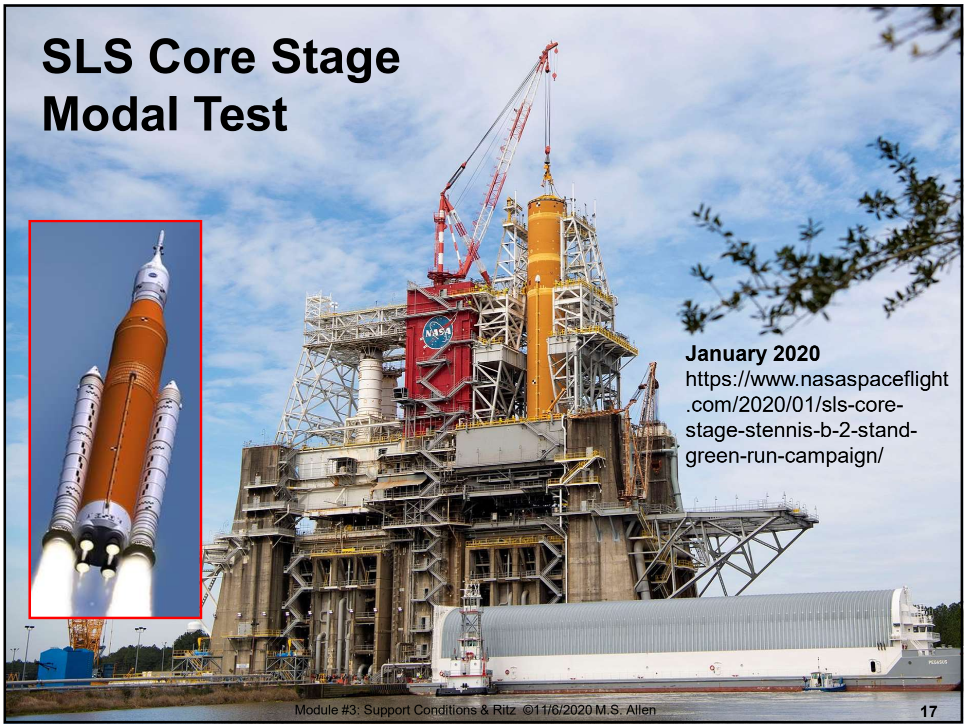
- At higher frequencies, fixed boundary conditions become even more difficult to create.
- “Everything turns to Jello above 2kHz.”
- Example, carbon fiber tube with axial resonance ~1500 Hz.



Modal tests usually mimic free-free boundary conditions since fixed conditions are more difficult to approximate experimentally.



- https://www.semanticscholar.org/paper/Testing-in-Aerospace-Research-Aircraft-Ground-at-Giclais-Lubrina/165433999cf435b030bd49182ce210a653239275/figure/1



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Support Conditions

- When can we neglect the bungee cords and/or support system in our modeling?

- The following comes from two papers:
  - T. G. Carne, D. Todd Griffith, and M. E. Casias, "Support conditions for experimental modal analysis," *Sound and Vibration,* vol. 41, pp. 10-16, 2007. (or) T. G. Carne, D. Todd Griffith, and M. E. Casias, “Support Conditions for Free Boundary-Condition Modal Testing,” 25th IMAC (IMAC XXV), Orlando, Florida, Feb. 19-22, 2007.
  - T. G. Carne & C. R. Dohrman, “Support conditions, their effect on measured modal parameters,” 16th IMAC, 1998.

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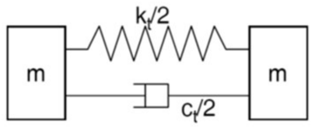
Simple Illustrative System

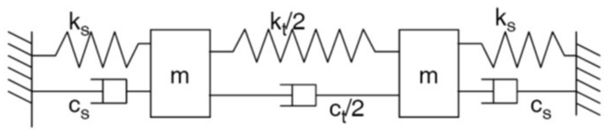
- Free Structure:



ϕ_1 = [1 1]^T $\quad \omega_1 = 0$

ϕ_2 = [1 -1]^T $\quad \omega_2 = \sqrt{k_t / m}$

- Supported Structure:



ϕ_1 = [1 1]^T

$$\omega_s = \sqrt{k_s / m}$$

ϕ_2 = [1 -1]^T

$$\omega_m = \sqrt{(k_t + k_s) / m}$$

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Error in Natural Frequencies

- Subscripts denote **true (t)**, **support (s)** and **measured (m)** natural frequencies:

$$\omega_m^2 = \omega_t^2 + \omega_s^2$$

$$\omega_t = \omega_m \left[1 - \frac{\omega_s^2}{\omega_m^2} \right]^{1/2}$$

- (Derivation on board)

$$\boxed{\frac{\Delta\omega}{\omega_m} = \frac{\omega_m - \omega_t}{\omega_m} \cong \frac{1}{2}\left(\frac{\omega_s}{\omega_m}\right)^2}$$

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Damping Analysis

- Easy to remember formula:

$$\zeta_m \omega_m = \zeta_t \omega_t + \zeta_s \omega_s$$

- Error Formula:

$$\zeta_t = \zeta_m \frac{\omega_m}{\omega_t}\left[1 - \frac{\omega_s}{\omega_m}\frac{\zeta_s}{\zeta_m}\right]$$

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Modal Parameter Sensitivity

- MDOF Systems can be treated using Modal Parameter Sensitivity Formulas (Ewins, "Modal Tesing," Research Studies Press, 2001.)

$$\frac{\partial}{\partial p}\left[[K]\{x\} - \omega^2[M]\{\ddot{x}\}\right]\{\phi\} = 0 \quad \Rightarrow$$

$$\frac{\partial \omega_r}{\partial p} = \frac{1}{2\omega_r}\{\phi_r\}^T\left(\frac{\partial[K]}{\partial p} - \omega_r^2\frac{\partial[M]}{\partial p}\right)\{\phi_r\}$$

- If the change is the addition of stiffness between the ith point and ground,

$$\Delta\omega_r = \frac{1}{2\omega_r}(\phi_r^i)^2\Delta k_i$$

- Where ϕ_r^i denotes the rth mode vector at the ith point.
  - Can this be extended to a continuous system?

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Example with 1-term Ritz Series

Equation Based on One-Term Ritz

- "Easy to Remember" Formula:

$$\omega_t^2 = \omega_m^2 - \omega_s^2 \frac{\psi(x_s)^2}{\psi_{\mathrm{rb}}(x_s)^2}$$

- Or Alternatively

$$\omega_t = \omega_m \sqrt{1 - \frac{\omega_s^2}{\omega_m^2} \frac{\psi(x_s)^2}{\psi_{\mathrm{rb}}(x_s)^2}}$$

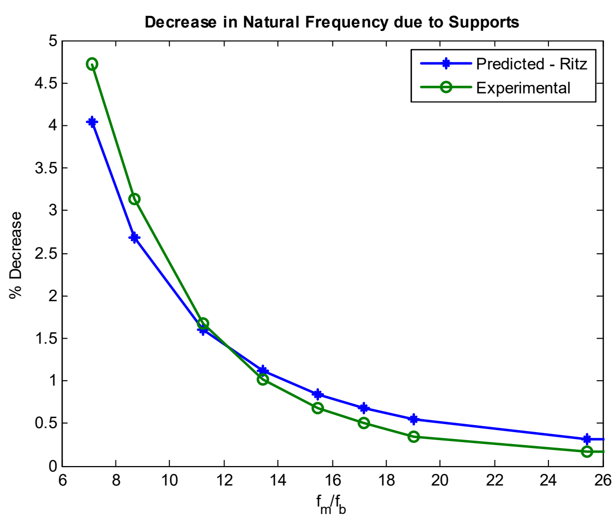
- This is based on these assumptions:
  - One-term Ritz model (mode shape of the mode of interest doesn't change)
  - One spring is attached at x_s causing the system to have a supported frequency ω_s

Experimental Application

Figure 3: Photo of Lightly Damped Aluminum Beam with Variable Length Elastic Supports

- Experiment from Carne, Griffith & Casias

Experimental Application



- Blue shows predicted change in natural frequencies using the formula from our 1-term Ritz analysis.
- That result also agrees well with the formula based on modal parameter sensitivity.

Damping Errors

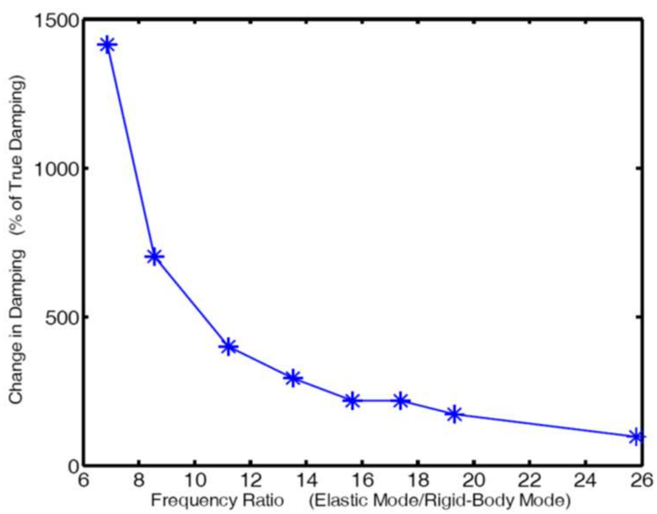


Figure 5: Observed Changes in Modal Damping as a Function of the Frequency Ratio

- Observed damping errors were very large and not predicted well by theory.
- Experimental result was found to be between the result predicted by structural and viscous damping models.

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Application to Wind Turbine Blade

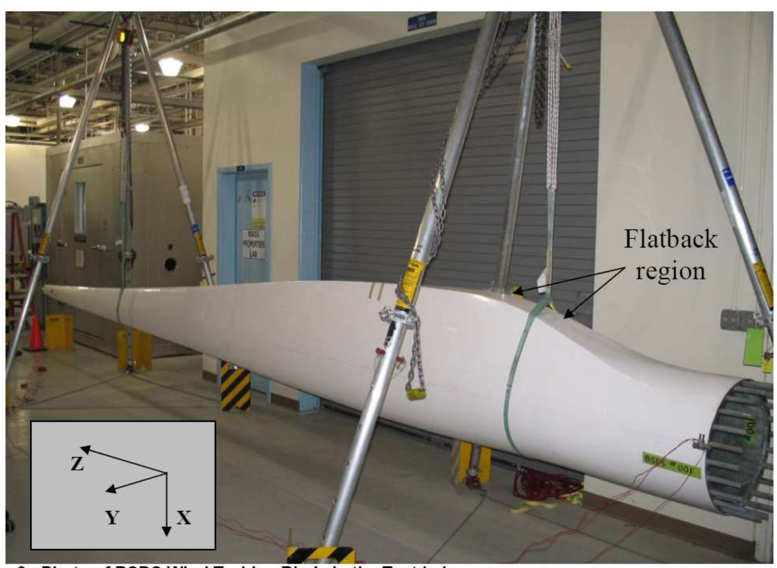

Figure 6: Photo of BSDS Wind Turbine Blade in the Test Lab

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Results

Table 2: Four Different Bungee Configurations for Supporting the Blade

Configuration Number	Support Characteristics		
	Description	Number of Loops	Motivation of Configuration
1	Bungees spaced 30 inches, either side, from CG	8,8	Low preload on each bungee loop of 20 pounds. Safe support design.
2		6,6	Slightly higher preload (25 pounds) reduces stiffness of bungee loops.
3	On the nodes of edgewise mode, 46 and 148 inches from CG	6,6	Moved to nodes of mode to reduced effect of bungee; preload changed
4		4,2	Reduced number of bungees to reduce support stiffness & balance preload

Table 3: Measured Modal Parameters for 4 Support Configurations for the Bending and Rigid-Body Modes

Config. No.	Rigid-Body Bounce Mode		First Edgewise Bending Mode				Ratio of Edgewise to Bounce Freqs.
	Freq. (Hz)	Damping Factor (%)	Freq. (Hz)	Increase from Conf. 4 (%)	Damping Factor (%)	Increase from Conf. 4 (%)	
1	4.72	4.2	16.38	2.	1.00	52	3.5
2	3.19	4.9	16.18	1.	0.80	21	5.1
3	5.59	5.2	16.09	0.1	0.73	10	3.1
4	1.28	3.2	16.07	-	0.63	-	12.5

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Report of Trip to NASA TDT Mtg

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NASA Engineering & Safety Center

- http://www.nasa.gov/offices/nesc/home/Overview.html
- Created after Challenger shuttle disaster, 1986.
- “NASA Engineering and Safety Center's (NESC) mission is to perform value-added independent testing, analysis, and assessments of NASA's high-risk projects to ensure safety and mission success. The NESC engages proactively to help NASA avoid future problems.”
- Sample Reports:
  - http://www.nasa.gov/offices/nesc/reports/index_new.html

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Space Shuttle Roll Out Example

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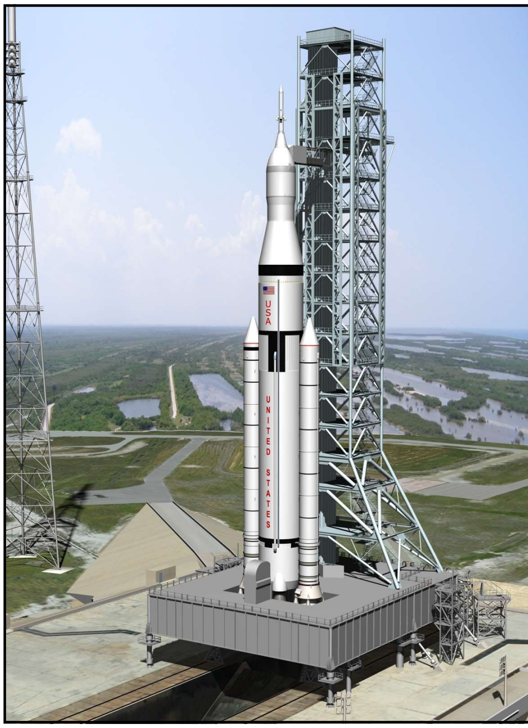

NASA ML & SLS

In 2017-18 Allen studied whether substructuring could be used to estimate the fixed-interface modes of the SLS from measurements on the Mobile Launcher.

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Finite Element Analysis

- Simple Matlab® code that creates finite element models of beam structures.
  - FEA_SimpleExamples.m

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Rocket Engine

- The J-2X is a liquid-fuel cryogenic rocket engine that was planned for use on NASA's Constellation program and Space Launch System. Built in the United States by Aerojet Rocketdyne (formerly, Pratt & Whitney Rocketdyne), the J-2X burns cryogenic liquid hydrogen & liquid oxygen propellants, with each engine producing 1,307 kN (294,000 lbf) of thrust in vacuum at a specific impulse (Isp) of 448 seconds (4.39 km/s).[2] The engine's mass is approximately 2,470 kg (5,450 Lb), significantly heavier than its predecessors.[2]