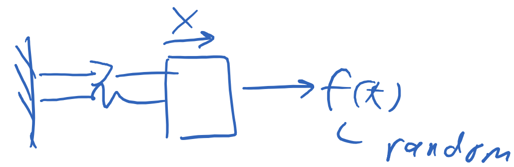
Module 6: Operational Modal Analysis

Monday, October 19, 2020 10:43 AM

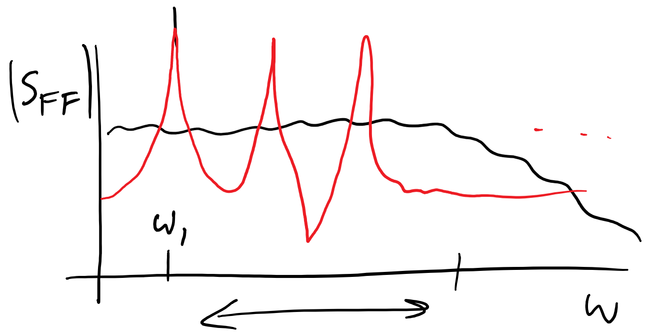


$$S_{xx} = E(XX^*)$$

$X = FFT(x)$

$$S_{xx} = E(\underline{H} F \underline{H}^* F^*)$$

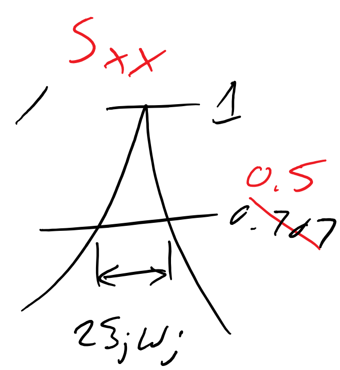
$$S_{xx} = |H|^2 E(FF^*)$$



$$\underline{S_{xx}} = \underline{|H|^2} S_{FF}$$

└ Looks like $|H(\omega)|^2$

(*) (if S_{FF} is flat
f(t) Broadband, Random
gaussian white noise)



$$S_{FF}(\omega) = \text{const}$$

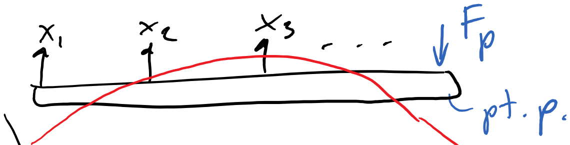
1.) Peak-picking

2.) Half-power BW $\rightarrow \zeta$ $(0.707 \rightarrow 0.5)$

$$|H(\omega)|^2 = \left| \frac{(1/m)}{\omega_n^2 - \omega^2 + i\omega 2\zeta\omega_n} \right|^2$$

$\omega = \omega_n$

3.) Mode Shapes



$$\begin{Bmatrix} S_{x_1x_1} \\ S_{x_2x_1} \\ S_{x_3x_1} \\ \vdots \end{Bmatrix} = E\begin{pmatrix} H_{1p} F_p H_{1p}^* F_p^* \\ H_{2p} F_p H_{1p}^* F_p^* \\ \vdots \end{pmatrix}$$

$$H_{jp} = \sum_{r=1}^{N} \frac{\phi_{jr}\phi_{pr}}{\omega_r^2 - \omega^2 + i\omega 2\zeta_r\omega_r}$$

r = mode #

$$= \{H(:,p)\} E(F_p F_p^*) H_{1p}^*$$

$$= (H(:,p))\, E(\ldots)$$

$\propto H(:,p)$ meas pts $[1, 2, 3 \ldots N_m]$

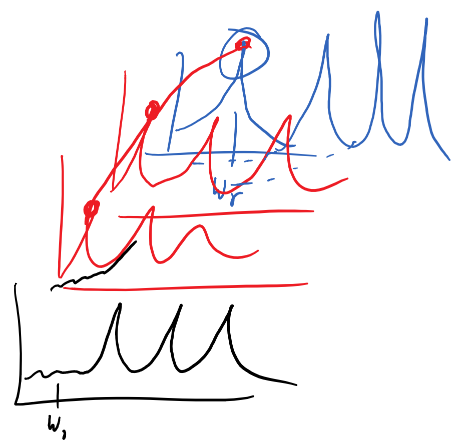
$$\{\downarrow\} = \left(\sum_r \frac{\{\phi\}_r \phi_{pr}}{\omega_r^2 - \omega^2 + i\omega 2\zeta_r \omega_r} \right) S_{F_p F_p} \left(\sum \frac{\phi_{1r}\phi_{pr}}{\omega_r^2 - \omega^2 \ldots} \right)$$

$$\begin{Bmatrix} S_{x_1 x_1} \\ S_{x_2 x_1} \\ S_{x_3 x_1} \\ \vdots \end{Bmatrix} = \left(\sum_{r=1}^{N} \frac{\{\phi_r\}\phi_{pr}}{\cancel{\omega_r^2} - \cancel{\omega^2} + i\omega 2\zeta_r \omega_r} \right) S_{FF} \left(\sum_{r=1}^{N} \frac{\phi_{1r}\phi_{pr}}{\cancel{\omega_r^2} - \cancel{\omega^2} + i\omega 2\zeta_r \omega_r} \right)$$

$$\{\phi_r\} = [\phi_{1r} \;\; \phi_{2r} \;\; \phi_{3r} \ldots]^T$$

$\omega = \omega_r$

$$\begin{Bmatrix} S_{x_1 x_1} \\ \vdots \end{Bmatrix} = \frac{\{\phi_r\}\phi_{pr} S_{FF} \phi_{1r}\phi_{pr}}{(i 2\zeta_r \omega_r^2)^2}$$



$$\boxed{\{S_{xx}\} = \alpha \{\phi_r\}}$$

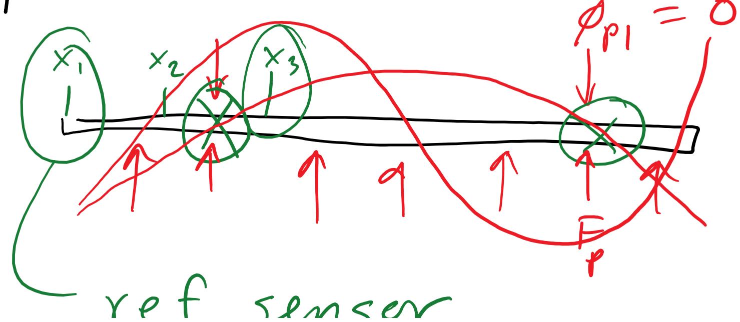
$$\begin{Bmatrix} S_{x_1 x_1} \\ S_{x_2 x_1} \\ \vdots \end{Bmatrix} = \{\phi\}_r \frac{S_{FF}\phi_{pr}^2 \phi_{1r}}{i\omega \zeta_r \omega_r^2}$$

S_{FF} – Force $\neq 0$ * @ $\omega = \omega_r$

ϕ_{pr} – modeshap * pt. force is app.

$\sum_j (S_{F_j F_j} \phi_{jr}^2)$

$\phi_{p1} = 0$



$\phi_{1r} \neq 0$ *

- ref. senser