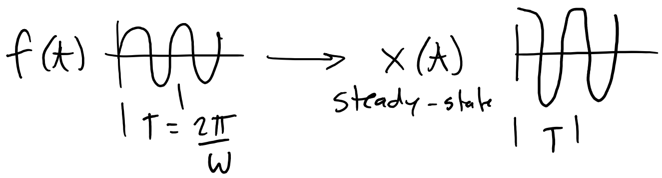
Lecture 3b - MDOF Forced Response

Friday, September 18, 2020 11:42 AM

SDOF System

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

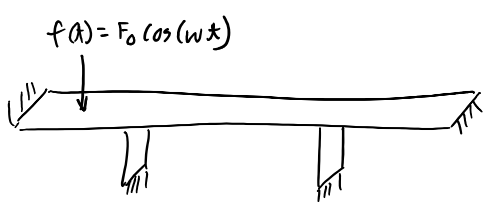


MDOF Systems:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = Re\left(\{F\}e^{i\omega t}\right)$$

assume: $\{x\} = Re\left(\{X\}e^{i\omega t}\right)$

Direct Solution: $\{X\} = \left(-\omega^2[M] + i\omega[C] + [K]\right)^{-1}\{F\}$



Modal Solution:

recall: $\left([K] - \omega^2[M]\right)\{\phi\} = 0 \longrightarrow \omega_1, \omega_2 \cdots \omega_N$, $[\Phi]$

$$[\Phi]^T[K][\Phi] = \begin{bmatrix} \omega_i^2 & 0 \\ 0 & \ddots \end{bmatrix}$$

$$[\Phi]^T[M][\Phi] = [I]$$

$\{x(t)\} = [\Phi]\{\eta(t)\} \longrightarrow$

$$\ddot{\eta}_j + 2\zeta_j\omega_j\dot{\eta}_j + \omega_j^2\eta_j = \{\Phi\}_j^T\{f(t)\}$$

$$= \{\Phi\}_j^T Re\left(\{F\}e^{i\omega t}\right)$$

$$= Re\left(F_{m,j}\, e^{i\omega t}\right)$$

as for SDOF System:

$$\eta_j(t) = Re\left(N_j e^{i\omega t}\right)$$

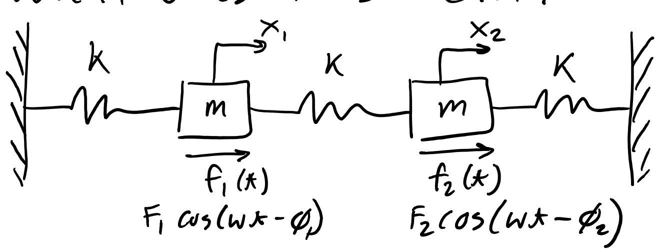
$$N_j = \frac{\{\Phi\}_j^T\{F\}}{\omega_j^2 - \omega^2 + i\omega 2\zeta_j\omega_j}$$

$$\begin{Bmatrix} X_1(t) \\ X_2(t) \\ \vdots \end{Bmatrix} = \begin{bmatrix} \{\Phi\}_1 & \{\Phi\}_2 & \cdots \end{bmatrix} \begin{bmatrix} Re(N_1 e^{i\omega t}) \\ Re(N_2 e^{i\omega t}) \\ \vdots \end{bmatrix}$$

$$\{x(t)\} = Re\left(\begin{Bmatrix} X_1 \\ X_2 \\ \vdots \end{Bmatrix} e^{i\omega t}\right) \qquad \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \end{Bmatrix} = [\Phi]\begin{Bmatrix} N_1 \\ N_2 \\ \vdots \end{Bmatrix}$$

$$\{X\} = \sum_{j=1}^{N} \frac{\{\Phi\}_j\left(\{\Phi\}_j^T\{F\}\right)}{\omega_j^2 - \omega^2 + i\omega 2\zeta_j\omega_j} = [H(\omega)]\{F\}$$

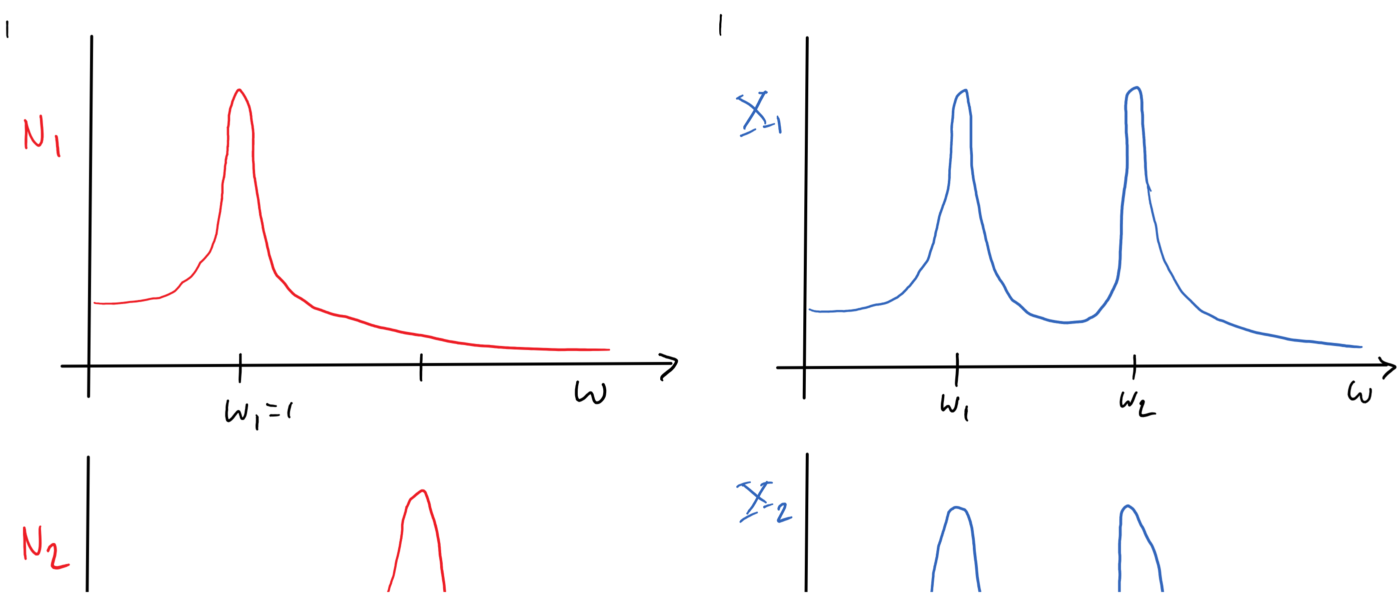
What does this mean?

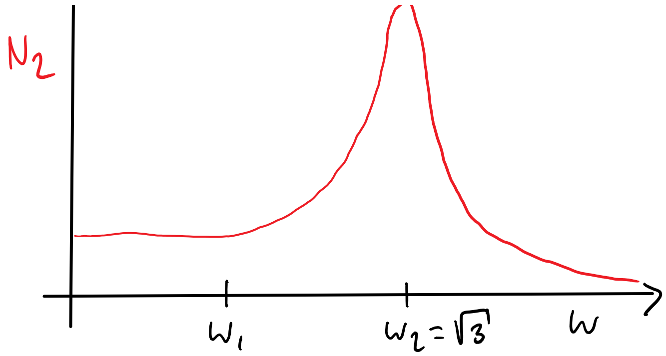


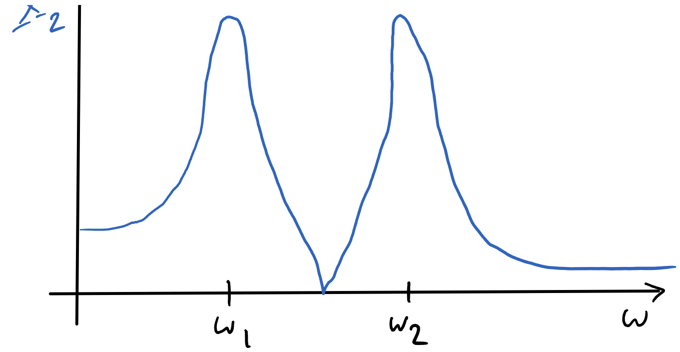
$$\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

$(k = \frac{1}{2},\ m = \frac{1}{2})$

$$[\Phi] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \omega_1 = 1,\ \omega_2 = \sqrt{3}$$







Near $w = w_1$

$$\{X\} = \sum_{j=1}^{N} \frac{\{\Phi\}_j \left(\{\Phi\}_j^T \{F\}\right)}{w_j^2 - w^2 + i w 2 \xi_j w_j}$$

$$= \frac{\{\Phi\}_1 \left(\{\Phi\}_1^T \{F\}\right)}{w_1^2 - w^2 + i w 2 \xi_1 w_1} + \frac{\{\Phi\}_2 \left(\{\Phi\}_2^T \{F\}\right)}{w_2^2 - w^2 + i w 2 \xi_2 w_2}$$