ME/EMA 540 – Fall 2020

Lab 1: Free Vibration of SDOF / MDOF Systems

Lab Report
You will be required to turn in a lab report summarizing your findings. Pay special attention to the questions in bold-italics throughout the lab handout. You do not need to discuss the hardware setup in detail or other items that were given in this handout.

Objective
The purpose of this lab is to understand the basic features of the free response of underdamped single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) systems. You will acquire measurements from a real system and estimate the system's parameters (e.g. ω_n, ζ, etc…) for both SDOF and MDOF systems. For the SDOF case, you will carefully compare the measured response to the analytical response that is expected. <u>Your key focus with that comparison is to seek to ascertain to what degree the analytical model is capable of describing the motion of the real system</u>.

In this lab, translational single DOF mass-spring systems will be examined. It is important to note that all physical systems contain some damping, even if no external damper is attached (you will not use the external damper in this lab). Figure 1 shows a photo of one of the rectilinear system that will be used for this experiment. Figure 2 shows a schematic.

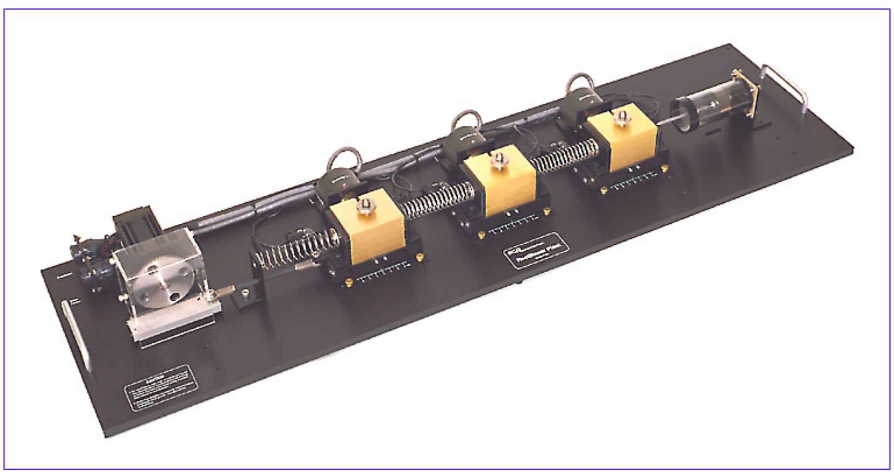


Figure 1: Rectilinear lumped-element dynamic system

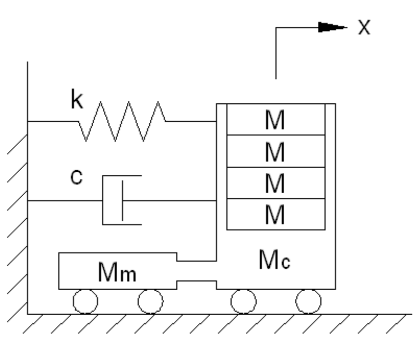


Figure 2: Translational system schematic

None of these parameters will be given to you except for the mass of the brass blocks (M = 0.59 kg) and even if we gave you number for the others they wouldn't be accurate. All of the other parameters must be determined from experimental data. Of course, one could determine the stiffness of the springs using statics and weigh the parts to estimate Mm and Mc the system, but there are some potential disadvantages to that approach. Instead, we shall use dynamic measurements to determine the unknown stiffness *k* and the mass of the entire cart Mm+Mc=m_1. Note that you will not connect the external dashpot, but the system still has some damping. You will also investigate a 2-DOF system whose schematic is shown below. Note that you should use identical springs so $k_1 = k_2$ and <u>you should not connect the dashpot</u>.

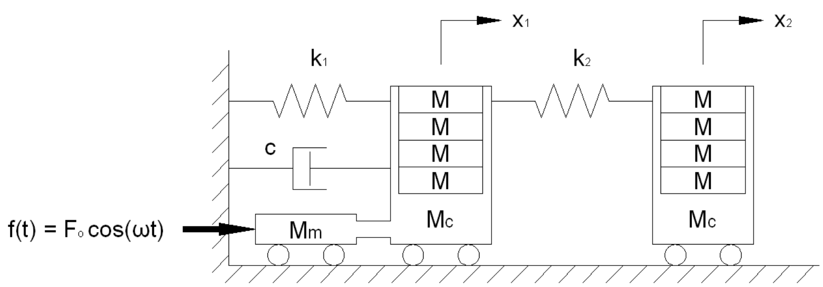


Figure 3: 2-DOF translational system schematic

The equations of motion for this system are:

$$M_1\ddot{x}_1 + \left(k_1 + k_2\right)x_1 - k_2 x_2 = f\left(t\right) \qquad (1)$$

$$M_2\ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0 \qquad (2)$$

TEAM MEMBER DUTIES (COVID-19 Adaptation)

Your lab groups consist of 2-3 people. Due to COVID-19, it will be necessary to maintain 6ft separation at all times. As a result, each team member will be given a distinct job. Only two team members will be allowed in the lab in 2163.

Team Member #1 – Run the computer including the B&K software. Send data to Team Member #3 as outlined below.

Team Member #2 – Handle the hardware, set initial conditions, change the number of masses connected, etc…

Team Member #3 (if applicable) – Connect with the lab via video conference to watch the procedure and discuss it / provide ideas to your team members. As they get their first data, they will send it to you. You should load it into Matlab immediately and start to create the plots mentioned, to check the quality of the data and for any issues. Provide feedback if any measurements seem questionable or if the settings (i.e. sample rate, initial amplitude) should be changed.

MEASUREMENT INSTRUCTIONS:

1) **SDOF SYSTEM:** Use the data acquisition system to capture and save the response of the system to various initial displacements from equilibrium. See the appendix for detailed instructions regarding how to setup the data acquisition system.
   a) Record the response of the system to initial displacements of about 1, 2 and 3 cm.
   b) Add a known mass to the system (m_2, so the total mass of the system is now m_1+m_2) and repeat the experiment once again for initial displacements of about 1, 2 and 3 cm.
      i) **ANALYSIS:** Find the natural frequency and damping ratio for the system with and without mass and use them to compute the mass, stiffness and damping (m, c and k) constants of the system. ***How uncertain are the natural frequency and damping ratio?*** Make sure your units are correct. ***Are your answers reasonable?***
      ii) **ANALYSIS:** Create a plot of the measured acceleration for at least one of the cases in (a) (no mass added) and compare it with the analytically reconstructed acceleration response (see detailed instructions in the Appendix for hints).
         (1) It is likely that, even after very carefully tuning your model, you may still not have perfect agreement between the test and model. Think carefully about the following questions to explore this.
         (2) ***How does the measured response compare with theory? Pay special attention to the damping envelope of the response and the frequency of the oscillations.***
         (3) ***If there are discrepancies, are these due to errors in your estimates for ζ and ω_n, or are you observing physics (e.g. Coulomb damping) that cannot be described by a linear, viscous model? Why?***
2) **MDOF SYSTEM:** Remove any masses from the carts and connect a second spring between the first and second carts. Make a note of what type of spring you used (thick or thin).
   a) Displace one of the carts about 2 cm while holding the other mass stationary and record the response. You should now see a multi-harmonic response since the system is now a 2-DOF system. Include a plot in your report.
   b) Displace both carts to the right by about the same amount and observe the response. If you displace them exactly in the pattern of the first mode of vibration, then they should oscillate in unison at the same frequency. Can you realize this experimentally? Record the best (closest to harmonic) response that you can obtain on the computer and save it so you can plot it for your lab report. ***Use your plot to estimate the mode shape and natural frequency of the first mode.***
   c) Displace the carts in opposite directions by about the same amount and record the response. ***Can you displace them such that they oscillate in a pure second mode? Record the best response that you can obtain. Is this more or less difficult than the previous case? Why or Why not? Use your plot to estimate the mode shape, natural frequency and damping ratio of the second mode.***
   d) **ANALYSIS:** Estimate the mode shape, natural frequency and damping ratio of each of the modes that you isolated in (b-c). Use those results to predict the acceleration response of the system to the initial condition given in (2.a) and overlay the prediction on the actual measured response. ***How do the two compare?***

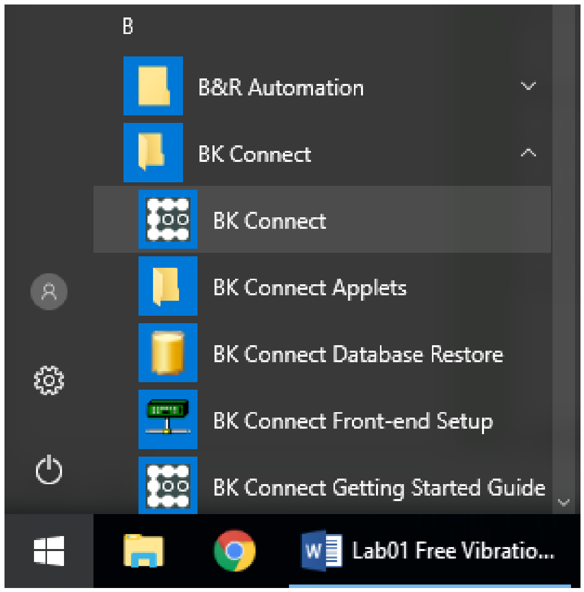
SUMMARY OF WHAT TO INCLUDE IN YOUR LAB REPORT:

1.) Plots
   a. comparing the response of the SDOF system with $m = m_1$ and $m = m_1+m_2$.
   b. comparing analytical response of SDOF (tuned to agree as closely as possible) to measured response with $m = m_1$.
   c. of MDOF system response to ICs $x_1 \approx 2$, $x_2 \approx 0$ with analytical response overlaid.
   d. of MDOF system response with ICs that you determined to isolate each of the two modes.
2.) Results:
   a. Identified natural frequencies and damping ratios for SDOF system with $m = m_1$ and $m = m_1+m_2$. Also discuss the uncertainty and how you estimated it.
   b. m_1, k_1, c_1 and c_2 computed from the modal parameters above
   c. identified natural frequencies and mode shapes of the MDOF system, found from the plots in (1d).
3.) Answers to the questions found in bold-italics throughout this lab report.
4.) You will also include any other plots, tables and analysis that is important to understanding the uncertainty in your results, why your results deviate / or do not deviate from the analytical model, etc…

APPENDIX

Data Acquisition Setup & Operation

a) Ensure the LAN-XI module is turned on and plugged in via Ethernet connection.
b) Connect one accelerometer to each of the carts. Make sure to connect them to the outside of the cart so that it will be easy to add and remove masses.
c) Connect the cables to the accelerometer and to channels 1 and 2 on the LAN-XI module.
d) Next open the BK Connect software from Start >>> BK Connect >>> BK Connect



e) Select the Time Data Recorder module on the left side of the UI
f) Ensure that the LAN-XI is visible in the Hardware Matrix.
g) In the HW Setup Table, navigate from 'Default' to the 'Channel' option in the drop down menu. Then check the box to enable the first two channels, change the Input Source from 'Direct' to 'CCLD', set the high pass filter to .7 Hz, assign the physical quantity as acceleration, and enter each accelerometers sensitivity, as read from the spec sheet in its box. (It should be around 50 mV / m/s^2)

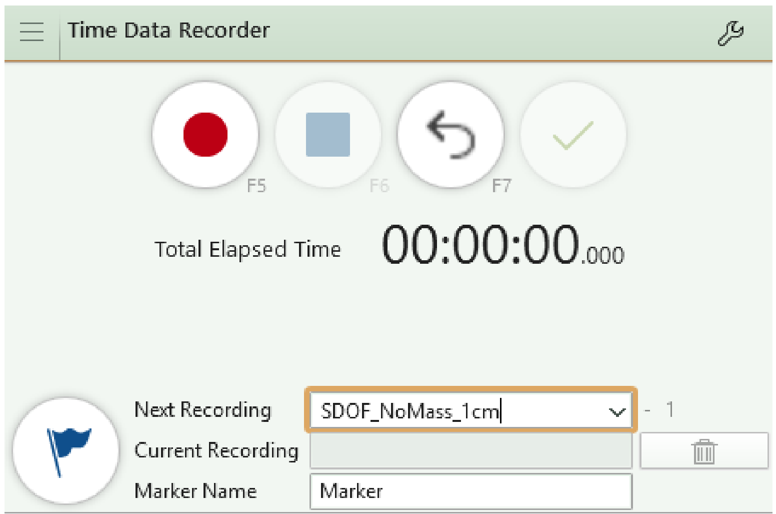
HW Setup Table (1/4) Channel

Enabled	Frame Module Chan...	Name	Input Source	Channel Type	High-pass Filter	Grounding	Physical Quantity	Channel Sensitivity
■						Groun		
☑	Input 1.1.1	Cart 1	CCLD	Input	0.7 Hz (0.1dB)	Grounded	Acceleration	mV/(m/s^2)
☑	Input 1.1.2	Cart 2	CCLD	Input	0.7 Hz (0.1dB)	Grounded	Acceleration	1000 mV/(m/s^2)
☐	Input 1.1.3	Signal 3	Direct	Input	DC	Grounded	Voltage	1 V/V
☐	Input 1.1.4	Signal 4	Direct	Input	DC	Grounded	Voltage	1 V/V

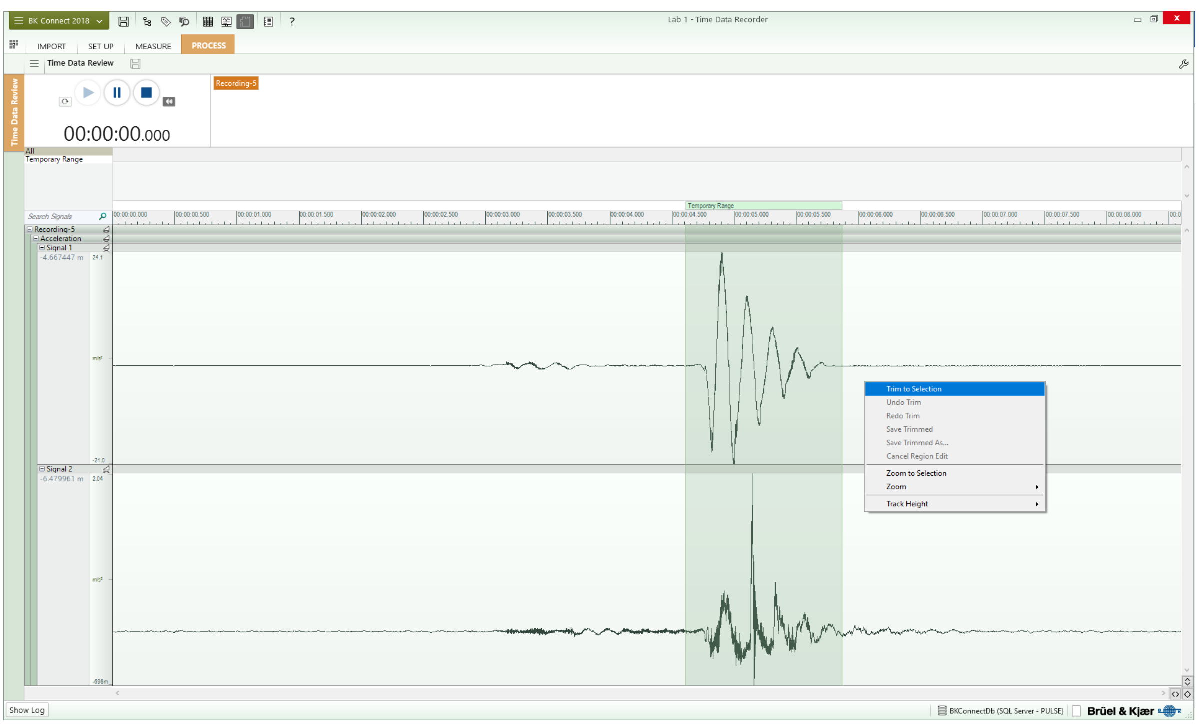
h) Set the System Frequency Range to 400 Hz.

i) On the right side, enter a name for the first recording as shown below.

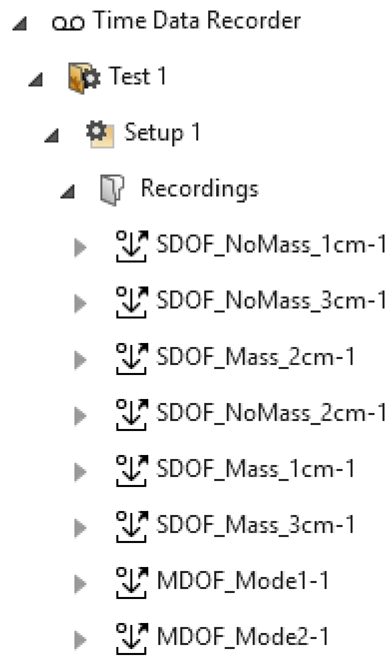
j) When ready to record, click the large button with the red dot and have one team mate displace the card a chosen initial displacement. When the cart motion has settled, press the button with the blue square to stop the recording.



k) Select the Process tab from the top of the Time Data Recorder interface. You should see at least one relatively clean oscillation. Click and drag on the signal of interest to select the region of the time history you wish to keep for analysis. Right click on the screen and select “Trim to Selection”.



l) Right click again and select 'Save Trimmed'
m) Repeat Steps i-j-k for all additional recordings.
n) To export the data, first toggle the Project Browser by pressing Alt+B. You should see something similar to what is shown below.



o) Right click on a recording and select 'Export data to File' and save the recording as a CSV file. Do this for all of your recordings. The CSV can be imported into MATLAB using its built-in Data Import tool.

SDOF SYSTEM ANALYSIS:

3) Load a pair of responses into MATLAB (e.g. the response when the mass is m_1 and a second when the mass is m_1+m_2). The files saved as mat files can be easily read in with the load('filename.mat') command.
   a) Import the file with the mass of the system being 'm_1'. The data from each channel will be put into vectors called 'Channel_#_Data'. The column headers for the data are stored in the struct 'Channel_#_Header'. The associated time vector can be calculated using the number of samples and the sampling frequency (both located in the 'File_Header' struct). Calculate the time vector by using the following equation. t =[0:NumofSamples-1]/(Sampling Frequency).
   b) Import the values from the second experiment for 'm_2' and repeat the process of extracting the time and position values into new vectors.
   c) Plot the responses using commands like: **>>plot(time1,xaccel1,time2,xaccel2);**. You can easily adjust for the initial condition offset by finding the resulting steady state displacement, using commands such as: >> **plot(time1+0.2,xposition1+0.3,…);**
4) Determine the natural frequency and damping ratio of the system in each configuration by measuring the period of the response and the decay envelope (log decrement). You may find it helpful to use the "**ginput**" command (e.g. [x,y] = ginput(2), etc…).

5) Solve for the parameters m_1 and k from the modal parameters (ω_1, ω_2 and ζ_1) that you have identified and the known mass m_2.
6) Solve for c_1 using the identified damping ratio ζ_1 (e.g. from the experiment with m_1). Repeat the solution using ζ_2 (from the experiment with m_1+m_2).
7) You were asked to compare the analytical response with one of your measured cases with m = m_1. Use the analytical free response equation in the text with the modal parameters that you've identified (ω_1 and ζ_1) to reconstruct the response and compare it with your measurements. Shift the phase of your initial condition to correspond as closely as possible with your experimental measurement. You may also need to adjust your estimates for (ω_1 and ζ_1). You may also adjust the initial condition for your analytical solution until you get the best agreement possible.

MATLAB Commands and Helpful Hints

Remember that MATLAB has a complete Help database that you can search by keyword or by using the MATLAB index.

For example: if you would like to see the see the help file on MATLAB's 'plot' command.

1. Open the Help database from the Help menu option

or

2. In MATLAB's Command window type 'help plot' at the prompt.

This can be done for any MATLAB commands. Also, the search option within the help database can lead to general help documents as well as help on specific commands.

Usual syntax issues:

Semi-colon: using a semi-colon after a command in the command window will suppress the display of the result of that command in the command window.

Colon: Used in element retrieval within a vector or matrix.

For example, if 'x' is a 1x10 vector:

a = x(:); - 'a' will be a new vector that contains all the elements of 'x'.

a = x(2:7); - 'a' will be a new vector that contains only the elements of 'x' that corresponds to the second through the seventh index.

For example, if 'M' is a 10x10 matrix:

B = M(:,:) ; - 'B' will be a new matrix that contains all the rows and columns of 'M'.

B = M(1:5,7:10); - 'B' will be a new matrix of size 5x4 that contains only the elements of 'M' that correspond to rows 1-5 and columns 7-10.

Apostrophe: ' – used to take the complex-conjugate transpose of a vector or matrix. If the vector or matrix is real-valued, then using an apostrophe just takes the transpose. To enforce just a transpose and not a complex-conjugate transpose, use: (.') or the dot-transpose after a matrix or vector.

Some useful command names to look at the help documentation for:

plot – a plotting command for vectors or matrices. Syntax: plot('xvector','yvector', 'options')

semilogy – a plotting command that plots the y-axis in a log scale
grid on; - turns a grid on the graph that allows for convenient visual interrogation
xlabel – allows the x-axis to be labeled
ylabel – allows the y-axis to be labeled
title – allows a title to be placed on the plot
abs- takes the absolute value of a variable or gives the magnitude
mean – takes the average of the components in a vector