Lecture 2c - Uncoupled EOM and Damping

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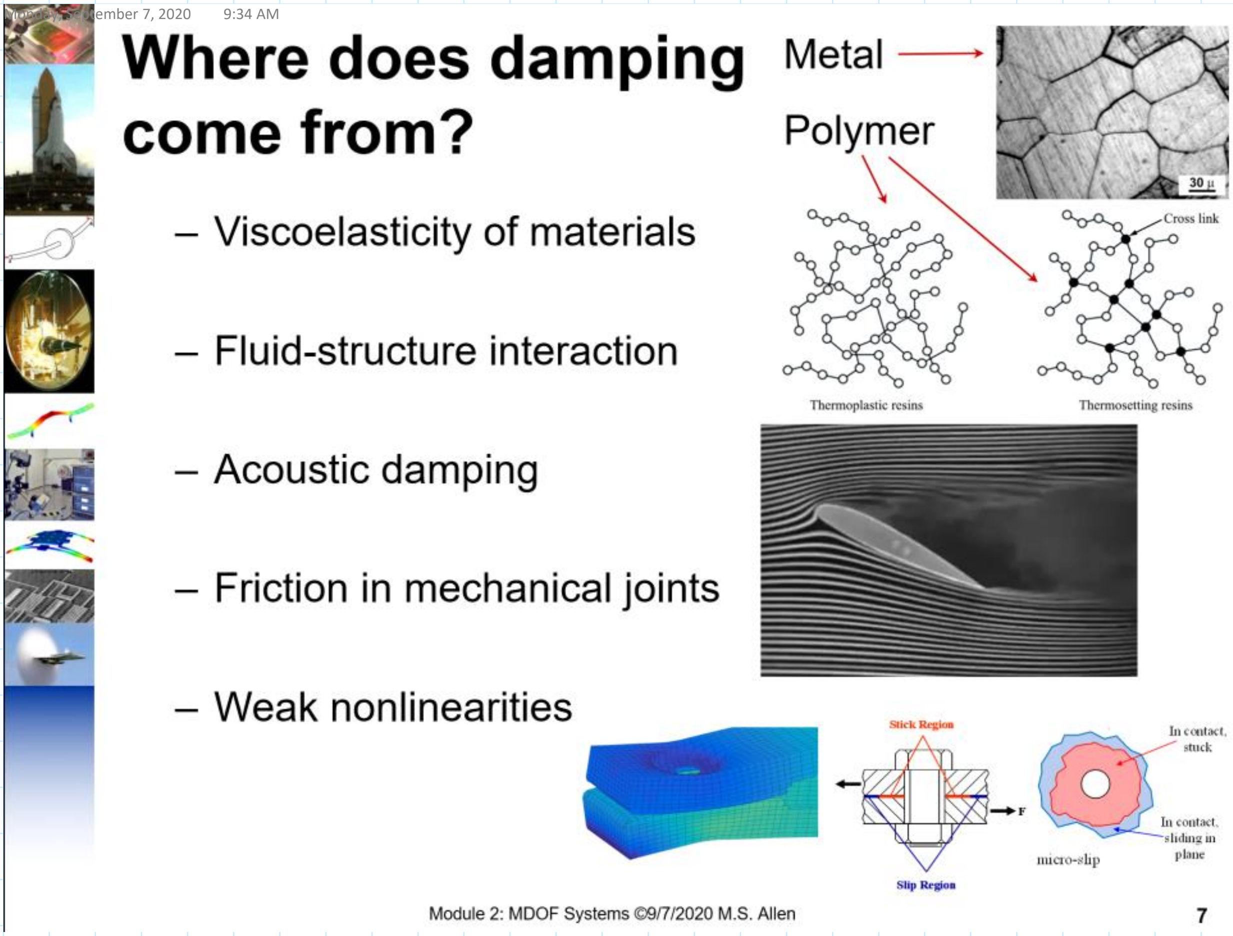


Remember the very important derivation covered in the last lecture:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f(t)\}$$

Let

$$\{x\}=[\Phi]\{\eta\}$$

Recall:

$$[\Phi]^T[M][\Phi]=[I],\quad [\Phi]^T[K][\Phi]=\left[\begin{smallmatrix}\ddots & & \\ & \omega_n^2 & \\ & & \ddots\end{smallmatrix}\right]$$

$$[\Phi]^T[M][\Phi]\{\ddot{\eta}\}+[\Phi]^T[C][\Phi]\{\dot{\eta}\}+[\Phi]^T[K][\Phi]\{\eta\}=[\Phi]^T\{f(t)\}$$

$$\ddot{\eta}_j+2\zeta_j\omega_j\dot{\eta}_j+\omega_j^2\eta_j=\{\Phi\}_j^T\{f(t)\}$$

iff

$$[\Phi]^T[C][\Phi]=\begin{bmatrix}2\zeta_1\omega_1 & 0 & 0 & \cdots\\ 0 & 2\zeta_2\omega_2 & 0 & \cdots\\ 0 & 0 & \ddots & \\ \vdots & \vdots & & \end{bmatrix}$$

When will this be true?

1.) Proportional Damping

If $[C]=\alpha[M]+\beta[K]$

Then the damping matrix will be diagonal, and you should easily be able to show that:

$$[\Phi]^T[C][\Phi] = \begin{bmatrix} \alpha+\beta\omega_1^2 & 0 & \cdots \\ 0 & \alpha+\beta\omega_2^2 & \cdots \\ \vdots & & \end{bmatrix}$$

So

$$2\zeta_j\omega_j = \alpha + \beta\omega_j^2$$

Choose α, β to get the desired ζ_j

- Most Common, esp. in FEA
- No physical basis → often non-physical...

2.) Light Damping Approximation:

Suppose we have $[C]$ *

$$[\Phi]^T[C][\Phi] = \begin{bmatrix} \hat{c}_{11} & \cancel{\hat{c}_{12}}^{\sim 0} & \cancel{\hat{c}_{13}}^{\sim 0} \\ \cancel{\hat{c}_{21}}^{\sim 0} & \hat{c}_{22} & \cdots \\ \cancel{\hat{c}_{31}}^{\sim 0} & \cancel{\hat{c}_{32}}^{\sim 0} & \ddots \\ \vdots & \vdots & \end{bmatrix}$$ Ignore off diagonals

Set: $\hat{c}_{11} = 2\zeta_1\omega_1$, $\hat{c}_{jj} = 2\zeta_j\omega_j \rightarrow \zeta_j = \dfrac{\hat{c}_{jj}}{2\omega_j}$

$$\boxed{\text{NOTICE: Use } \hat{C} = [\Phi]^T[C][\Phi] \text{ not } [C]\text{!!!}}$$

3.) Modal Damping

Damping Ratios ζ_j are given

- Based on past experience
- Measured in test
- Use worst-case (lowest) damping
- etc

Examples:

past experience
- Steel Material (no joints) $\zeta_j = 0.0001 - 0.0005$
- Composite Material $\zeta_j \cong 0.01$

worst case
- Aircraft flutter FAA mandates $\zeta_j = 0.03$ maximum until data is available.

Most common approach

4.) Material Damping (i.e. visco-elastic materials)

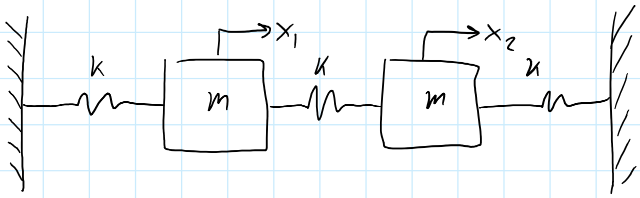
$$[\hat{K}] = [K](1 + i\gamma)$$

- produces damping effect in frequency-domain
- See Ginsberg book, EMA 630 (Lakes), etc...

All cases assume $[C]$ is diagonalized by $[\Phi]$ → uncoupled damping!

→ Ginsberg Ch. 10 relaxes this assumption.

Matlab Example: "Mod2_MDOF_FreeRespEx.m"



$m = 1, \quad k = 1$

Simulate free-response with proportional and non-proportional damping.