$$\underset{1\times 2}{\{\phi_1\}^T}\underset{2\times 2}{[M]}\underset{2\times 1}{\{\phi_1\}} = \mu_1 \qquad \{\Phi_1\} = \frac{\{\phi_1\}}{\sqrt{\mu_1}}$$

$$[\Phi] = [\{\Phi_1\}\{\Phi_2\}\cdots]$$

$\{x\} = [\Phi]\{\eta\} \longrightarrow$ plug in to EOM

$$[M][\Phi]\{\ddot{\eta}\} + [C][\Phi]\{\dot{\eta}\} + [K][\Phi]\{\eta\} = \{F\}$$

pre-mult by $[\Phi]^T$

$$\{\ddot{\eta}\} + [\Phi]^T[C][\Phi]\{\dot{\eta}\} + [\diagdown\omega_n^2\diagdown]\{\eta\} = [\Phi]^T\{F\}$$

if some assumptions about $[C]$ are met

$$[\Phi]^T[C][\Phi] = \begin{bmatrix} 2\zeta_1\omega_1 & 0 & 0 \\ 0 & 2\zeta_2\omega_2 & 0 \\ 0 & 0 & \ddots \end{bmatrix}$$

$$\ddot{\eta}_1 + 2\zeta_1\omega_1\dot{\eta}_1 + \omega_1^2\eta_1 = \{\Phi_1\}^T\{F\}$$ only η_1

$$\ddot{\eta}_2 + 2\zeta_2\omega_2\dot{\eta}_2 + \omega_2^2\eta_2 = \{\Phi_2\}^T\{F\}$$

$\vdots$

modal eq. of motion for the rth mode

2nd

Force applies to all modes

SDOF system

gen. sol

$$\eta_r(t) = \mathrm{Re}(A_r e^{\lambda_r t})$$

$$\lambda_r = -\zeta_r\omega_r \pm i\,\omega_{dr}$$

$$\omega_{dr} = \omega_r\sqrt{1-\zeta_r^2}$$

$\eta_1(t),\ \eta_2(t)\ldots$

$$\{x\} = [\Phi]\begin{Bmatrix}\eta_1(t)\\ \eta_2(t)\\ \vdots\end{Bmatrix}$$

"Mod2_MDOF_~~...~~FreeResp Ex.m"

physical coord.

resp. in modal coord

VI.C Initial Cond on Modal Coord

$$\underbrace{\{\eta\}}_{\eta(0)} = [\Phi]^{-1} \underbrace{\{x\}}_{x(0)} = [\Phi]^{T} [M] \underbrace{\{x\}}_{x(0)}$$