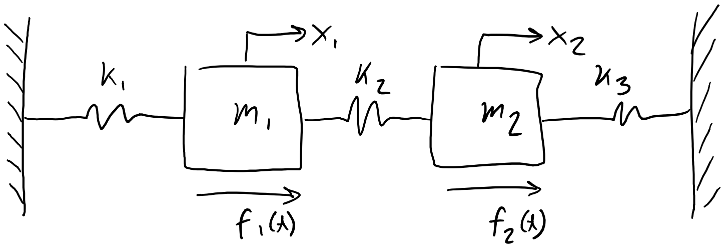
Lecture 2 - MDOF Systems & EVP

Tuesday, September 1, 2020 1:30 PM



EOM must have the form:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & (k_2+k_3) \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

- Can't solve: coupled ode's

Assume Solution:

$$\{x\} = Re\left(\{\phi\} e^{i\omega t}\right) = \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} \qquad \{\phi\} = \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix}$$

$$\left[[K] - \omega^2 [M]\right]\{\phi\} = 0$$

1.) Solve by hand (EMA545)
- Determinant → char. eqn. - ω_r's
- Eigenvectors/mode shapes $\{\phi\}_r$

2.) Solve in Matlab (this class)

- Factor out k's and m's so we have only numbers
- i.e. above let: $k_1 = k_2 = k_3 = k$, $m_1 = m_2 = m$

$$\rightarrow \left[\begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} - \omega^2 \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\right]\{\phi\} = 0$$

$$\frac{1}{\cancel{k}}\left[\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{K_n} \cancel{k} - \frac{\omega^2 m}{k}\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{M_n}\right]\{\phi\} = 0 \; \frac{1}{k}$$

- numerical

Now solve:

$$[K_n - \lambda M_n]\{\phi\} = 0 \qquad \lambda = \frac{\omega^2 m}{k}$$

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[phi, lam] = eig(Kn, Mn);
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$$lam = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \rightarrow$$ `Wn = sqrt(diag(lam));` $\left(* \sqrt{\frac{K}{m}}\right)$

For this problem, $\lambda_1 = 1$, $\lambda_2 = 3$ $\quad \phi_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$ $\phi_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

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$\omega_1 = \sqrt{\frac{K}{m}}$ $\quad \omega_2 = \sqrt{3}\sqrt{\frac{K}{m}}$

Two key properties of modes:

$$\{\phi\}_j^T [M] \{\phi\}_n \begin{array}{l} = 0 \quad \text{if } n \neq j \\ = \mathcal{M}_j \quad \text{if } n = j \end{array}$$

$$\{\phi\}_j^T [K] \{\phi\}_n \begin{array}{l} = 0 \quad \text{if } n \neq j \\ = \mathcal{M}_j \omega_j^2 \quad \text{if } n = j \end{array}$$

Example: 2-24 Bridge, Airplane