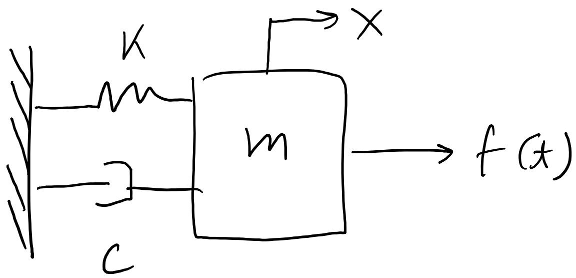
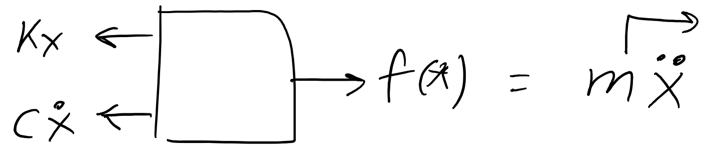
ME/EMA 540 Lecture 1 - SDOF System Response

Monday, August 31, 2020 4:36 PM



$$\Sigma \vec{F} = m\vec{a}$$



$$m\ddot{x} + c\dot{x} + Kx = f(t)$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{K}{m}x = f(t)/m$$

$$\frac{c}{m} = 2\zeta\omega_n \qquad \frac{K}{m} = \omega_n^2 \qquad \omega_n = \sqrt{\frac{K}{m}}$$

Undamped $(c=0)$ Free Response $(f(t)=0)$

$$\ddot{x} + \omega_n^2 x = 0$$

$$x(t) = c_1\cos(\omega_n t) + c_2\sin(\omega_n t)$$

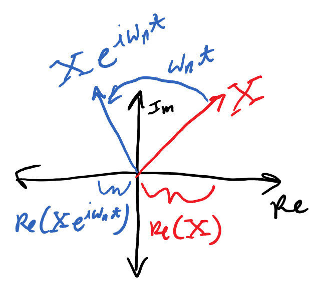
or

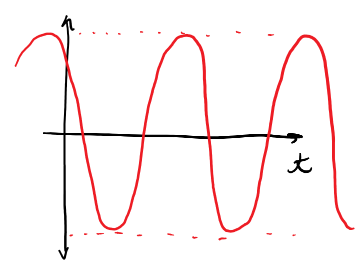
$$x(t) = \mathrm{Re}\left(X e^{i\omega_n t}\right)$$

$\mathrm{Re}(X)$, $\mathrm{Im}(X)$ from init. conds.

Damped Free Response

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$





$$x(t) = \mathrm{Re}\left(X e^{\lambda t}\right)$$

$$\dot{x}(t) = \mathrm{Re}\left(\lambda X e^{\lambda t}\right)$$

$$\ddot{x}(t) = \mathrm{Re}\left(\lambda^2 X e^{\lambda t}\right)$$

plug in above

$$\mathrm{Re}\left(\lambda^2 X e^{\lambda t}\right) + \underbrace{2\zeta\omega_n}_{\text{real \#'s}}\mathrm{Re}\left(\lambda X e^{\lambda t}\right) + \underbrace{\omega_n^2}_{\text{real \#'s}}\mathrm{Re}\left(X e^{\lambda t}\right) = 0$$

$$\mathrm{Re}\left(\left[\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2\right] X e^{\lambda t}\right) = 0$$

$$Re\left(\underbrace{\left[\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2\right]}_{=0} X e^{\lambda t}\right) = 0$$

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

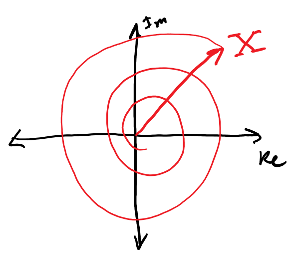
$$\lambda = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4(1)\omega_n^2}}{2(1)}$$

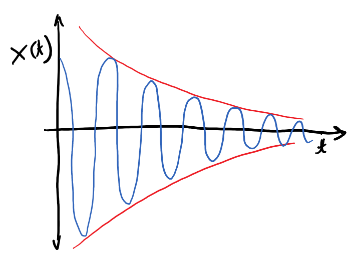
$$\lambda = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$\boxed{\lambda = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}}$$

define $\quad \omega_d = \omega_n\sqrt{1-\zeta^2}$

$$x(t) = Re\left(X e^{\lambda t}\right)$$
$$= Re\left(X e^{(-\zeta\omega_n + i\omega_d)t}\right)$$
$$= Re\left(X \underbrace{e^{-\zeta\omega_n t}}_{\text{Amplitude}} \underbrace{e^{i\omega_d t}}_{\text{oscillation (same as above)}}\right)$$





In this class, we can check most of our answers using numerical integration. (See slides on course website for a review.) It also helps us to solve nonlinear problems or other problems that don't have analytical solutions.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t)/m$$

define $\quad \underbrace{z = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}}_{2\times1\text{ vector}} \longrightarrow \dot{z} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix}$

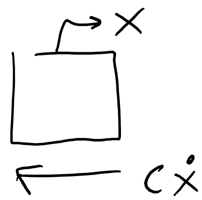
$$\dot{z} = \begin{Bmatrix} \dot{x} \\ -2\zeta\omega_n\dot{x} - \omega_n^2 x + f(t)/m \end{Bmatrix} = \{f(z)\}$$

$$\dot{z} = \begin{Bmatrix} \underbrace{\dot{x}}_{z_2} \\ -2\zeta\omega_n \underbrace{\dot{x}}_{z_2} - \omega_n^2 \underbrace{x}_{z_1} + f(t)/m \end{Bmatrix} = \{f(z)\}$$

define function $f(z)$ in Matlab to solve.

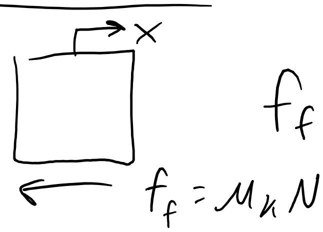
Damping

Viscous:



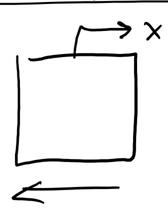
$$f_d = -c\dot{x}$$

Coulomb Friction: (See video for review)



$$f_f = -\mu_k N \,\mathrm{sign}(\dot{x})$$

Aerodynamic Drag:



$$f_d = \frac{1}{2} \underbrace{C_d}_{\text{drag coef.}} \underbrace{A_p}_{\substack{\text{projected area} \\ (m^2)}} \underbrace{\rho_f}_{\substack{\text{fluid density} \\ (kg/m^3)}} \dot{x}^2 \,\mathrm{sign}(\dot{x})$$

See: ode45_example_SDOF.m