Introduction to Experimental Modal Parameter Identification & AMI

ME / EMA 540

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Modal Parameter Identification

- Basic objective of Experimental Modal Analysis is to fit the response to the following mathematical form in order to identify the modal parameters ω_r , ζ_r and ϕ_r

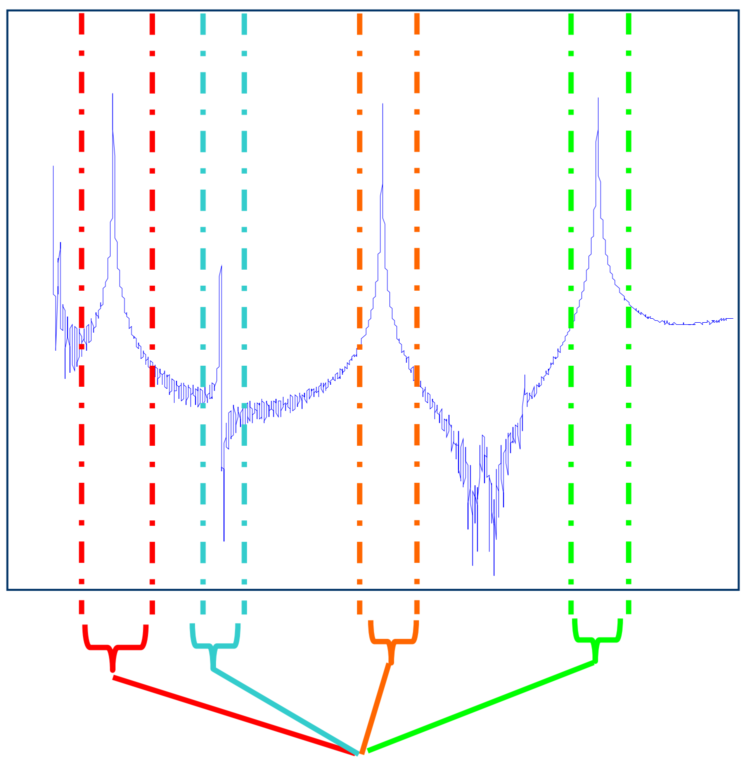
$$[\mathbf{H}] = \sum_{r=1}^{N} \frac{\{\phi\}_r \{\phi\}_{p,r}^{\mathrm{T}}}{\omega_r^{\ 2} - \omega^2 + \mathrm{i}\omega 2\zeta_r \omega_r}$$

- The most basic approach involves adjusting these parameters until the least-squares error between the measured response and the analytical representation is minimized.

$$[\mathbf{H}]_{measured} - \sum_{r=1}^{N} \frac{\{\phi\}_r \{\phi\}_{p,r}^{\mathrm{T}}}{\omega_r^{\ 2} - \omega^2 + \mathrm{i}\omega 2\zeta_r \omega_r} = error$$

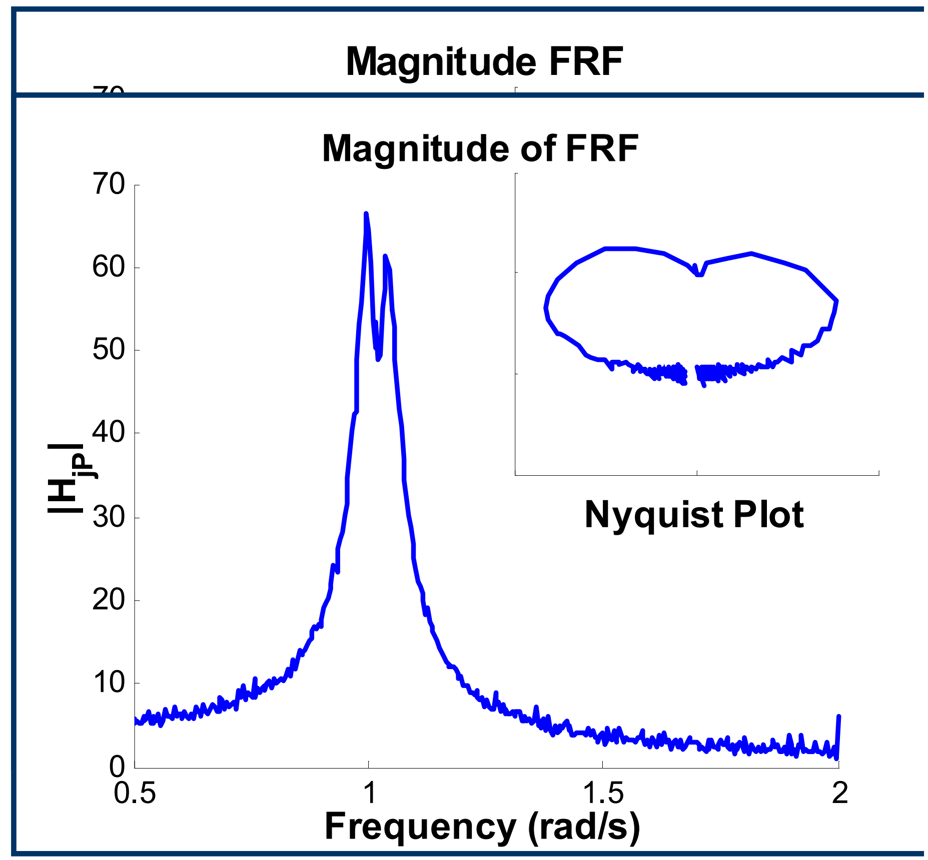
SDOF vs. MDOF Parameter Identification

- SDOF
  - Does not account for overlapping modes



SDOF Fits

- MDOF

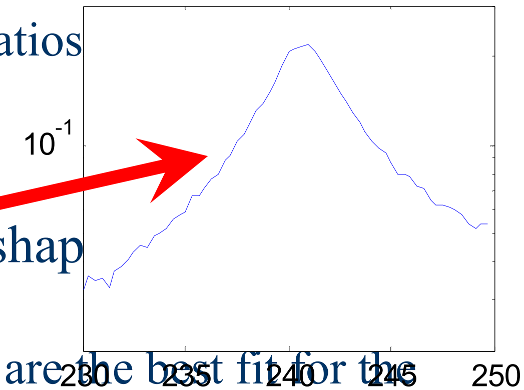


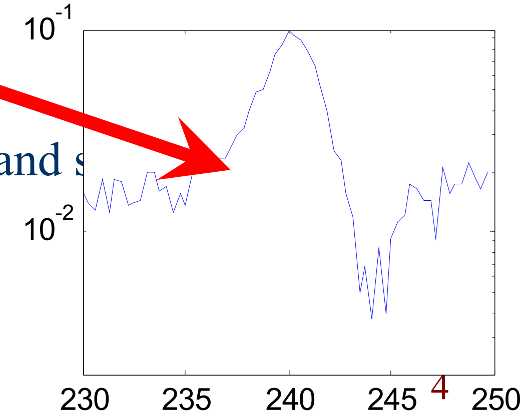
MDOF Fit

Global vs. Local Identification

- Local Modal Parameter Identification:
  - Identify ω_r, ζ_r and an element of ϕ_r from each response independently, and post process to find a global set of natural frequencies and damping ratios.
    - Advantage: Close control over the quality of fit in each FRF.
    - Disadvantage: Labor intense for complex structures.
    - Disadvantage: Natural Frequencies and Damping Ratios using all available information.

- Global Modal Parameter Identification:





ode shap

ratios are the best fit for the

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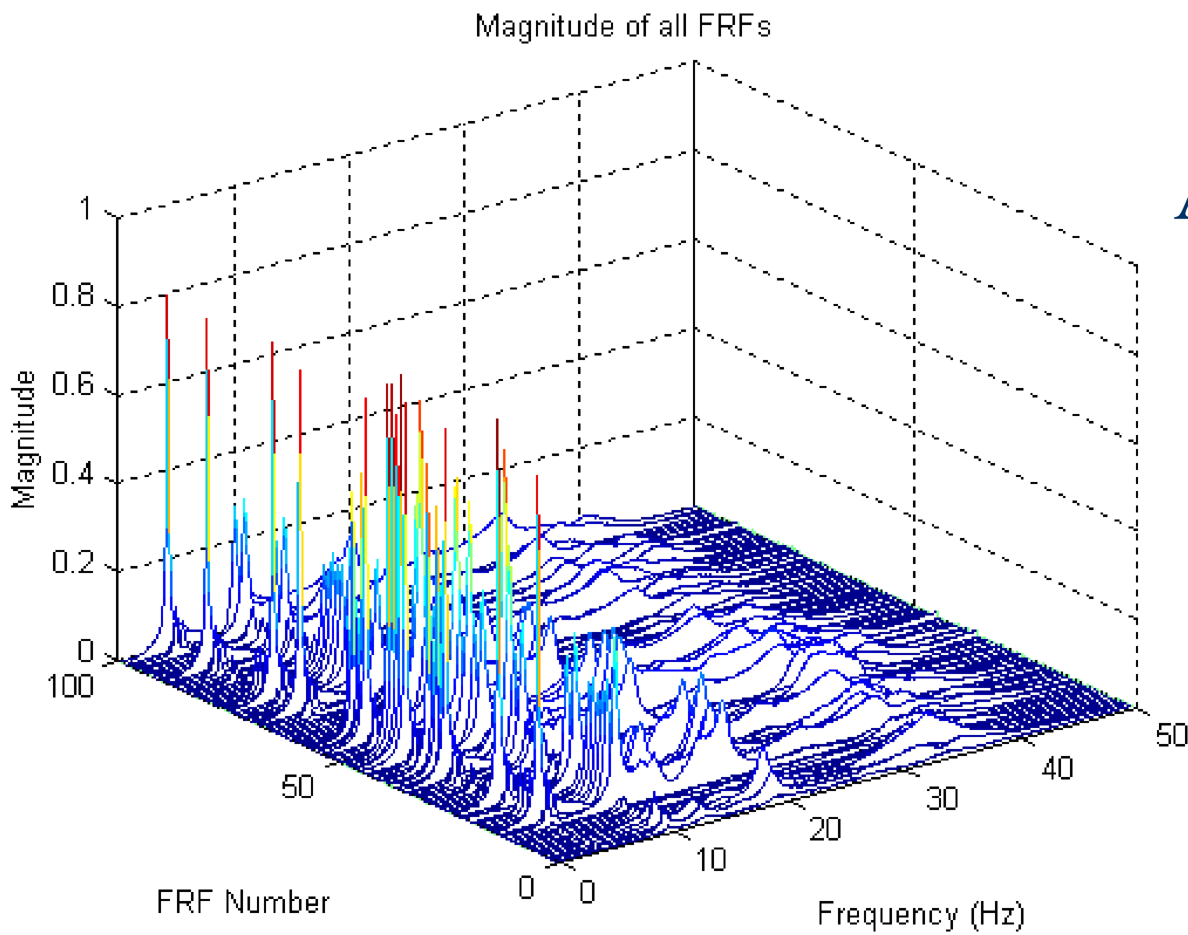
Global Identification: Schematic

- FRF Matrix for Highway Bridge



Composite FRF:

Average of the <u>Magnitude</u> of all FRFs

Least Squares Modal Parameter Ident.

- Consider a state space, SIMO FRF

$$[H(\omega)] = \sum_{r=1}^{N} \frac{\{\phi_r\}\{\phi_r\}^T}{\omega_r^2 - \omega^2 + 2\zeta_r\omega_r i\omega} \Rightarrow \{H_P(\omega)\} = \frac{\sum_{k=1}^{N_n} \{B_k\}(i\omega)^k}{\sum_{k=1}^{N_d} a_k (i\omega)^k}$$

- Linear Least Squares Form (single frequency):

$$\Big[W(\omega)\{H_p(\omega)\}[\Omega_0(\omega) \cdots \Omega_{N_d}(\omega)] \vdots$$

$$\vdots - W(\omega)[\Omega_0(\omega)[I_{N_o}] \cdots \Omega_{N_n}(\omega)[I_{N_o}]]\Big]\{\alpha\beta\} = \{0\}$$

$$\{\alpha\beta\} = \left[a_0\, a_1 \cdots a_{N_d}\ \{B_0\}^T\ \{B_1\}^T \cdots \{B_{N_n}\}^T\right]^T$$

Least Squares MPI

- Form Normal Equations (similar to the method in [Guillaume et al 1998]):

$$\mathrm{Re}\left\{[J]^H [J]\right\}\{\alpha\beta\} = \begin{bmatrix} [A_{11}] & [A_{12}] \\ [A_{12}]^H & [A_{22}] \end{bmatrix}\{\alpha\beta\} = 0$$

- Eliminate Numerator Coefficients:

$$\left[[A_{11}] - [A_{12}][A_{22}]^{-1}[A_{12}]^H\right]\{\alpha\} = [M]\{\alpha\} = \{0\}.$$

- When a single mode is fit, the largest matrix to be inverted is 2x2.

Model Order Determination

Sample Stabilization Diagram

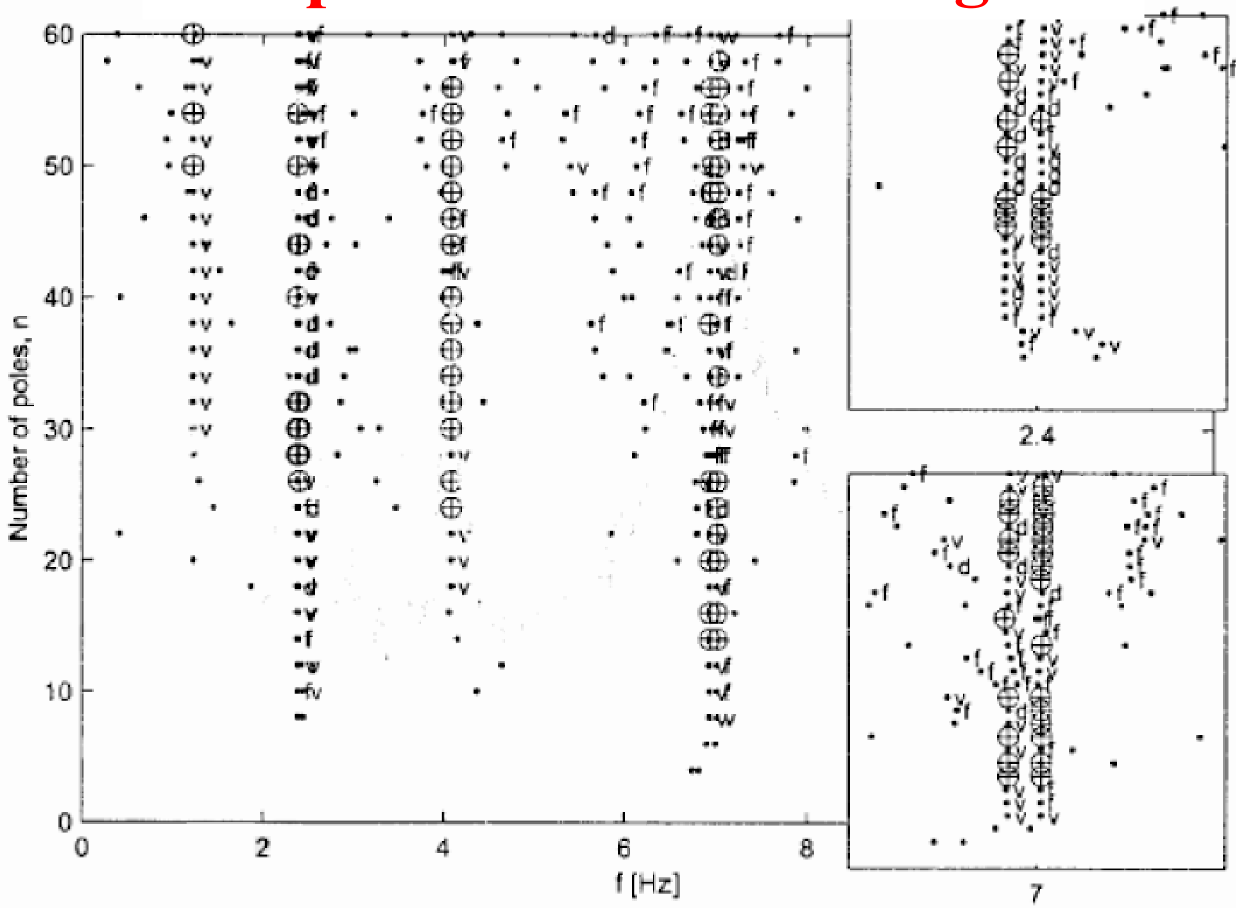


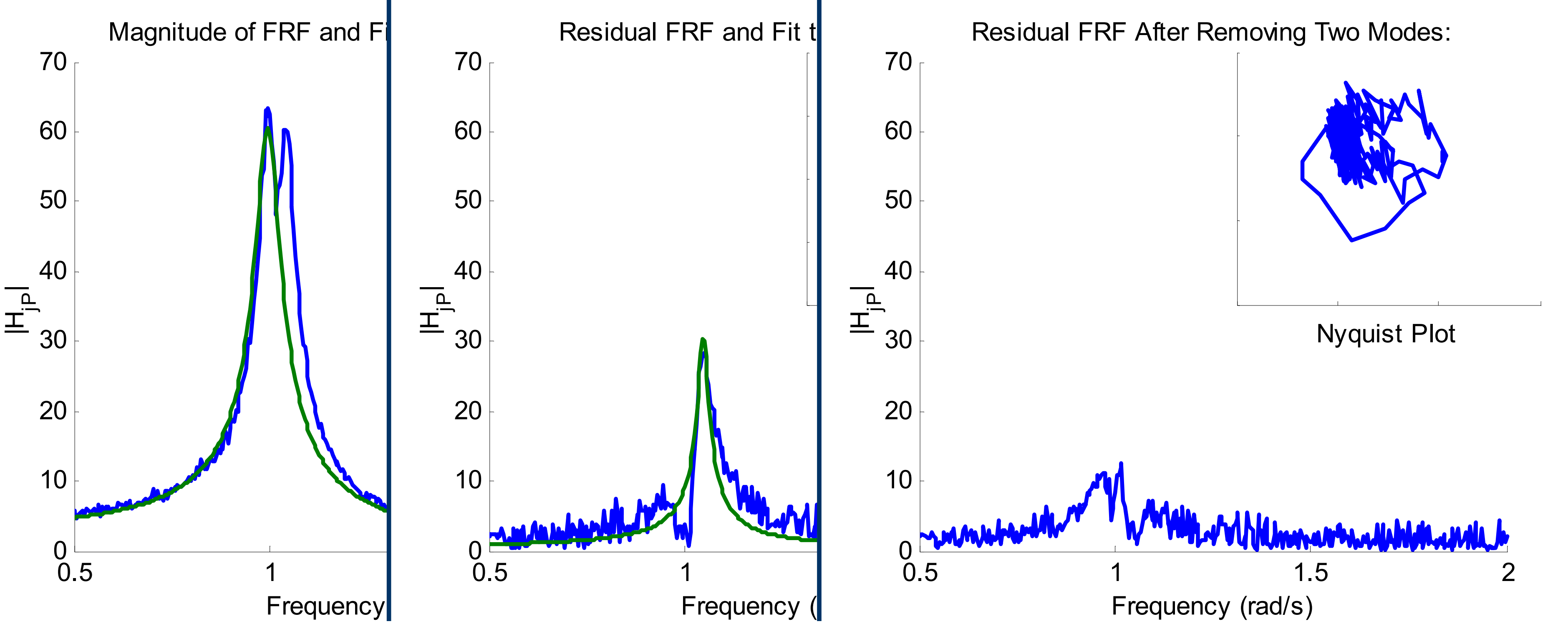
Diagram above from: [Peeters & De Roeck 1999]

- **Determining Model Order:**

1. Try various model orders.
2. Plot the natural frequencies obtained versus model order.
3. Look for frequencies that are consistently obtained.

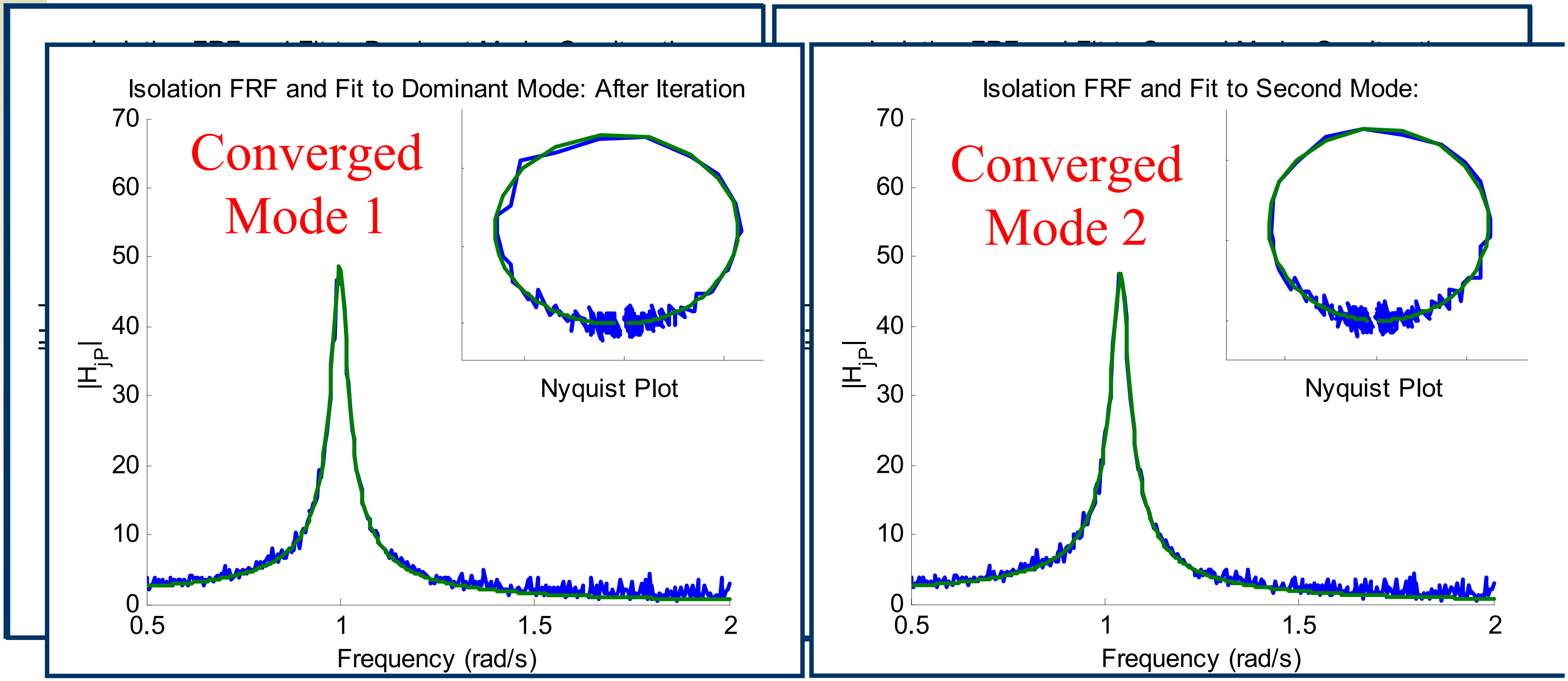
The Algorithm of Mode Isolation

- The Algorithm of Mode Isolation (AMI) offers a different approach. [Drexel & Ginsberg 2001, Zaki 2002]
  - Modes are successively identified and subtracted from the experimental data set.



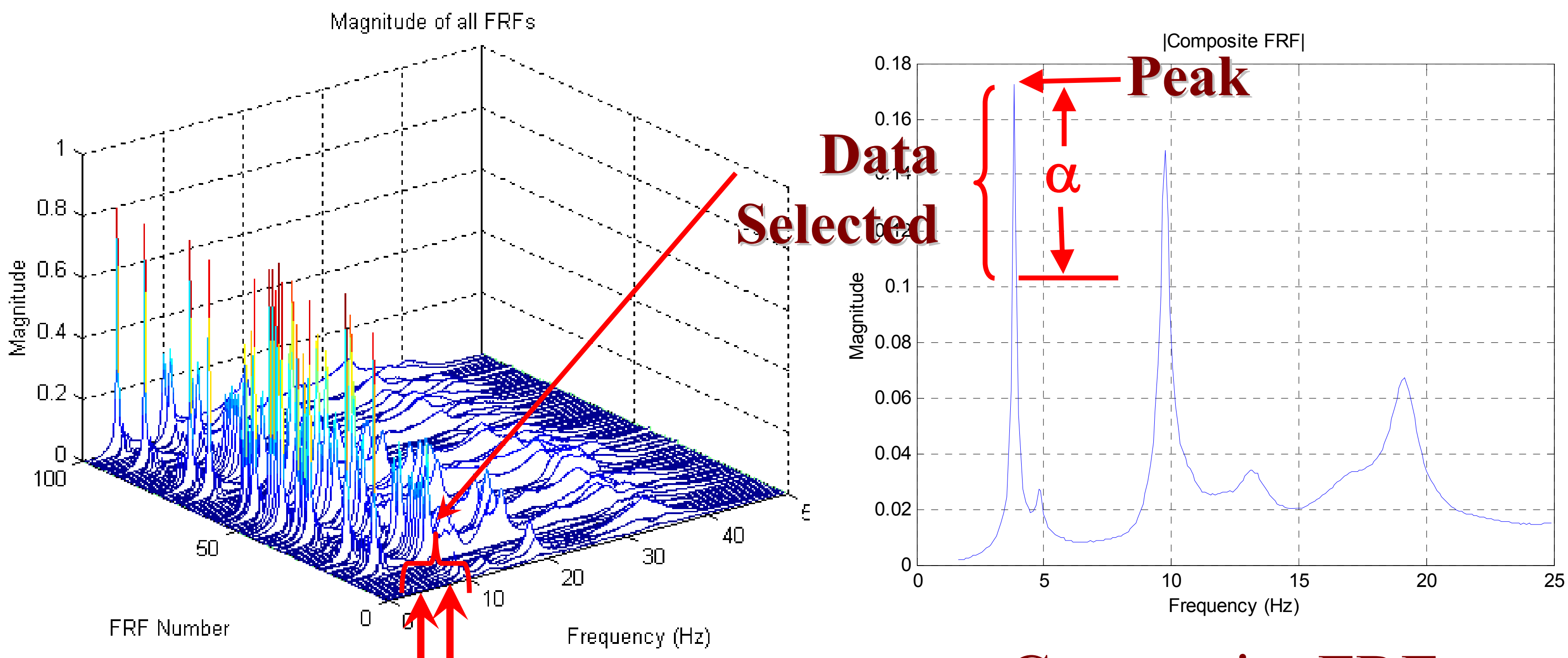
AMI – Isolation Stage

- Modes are then refined through an iterative procedure to account for overlapping contributions.



Global AMI

- Global AMI processes all available FRFs simultaneously.

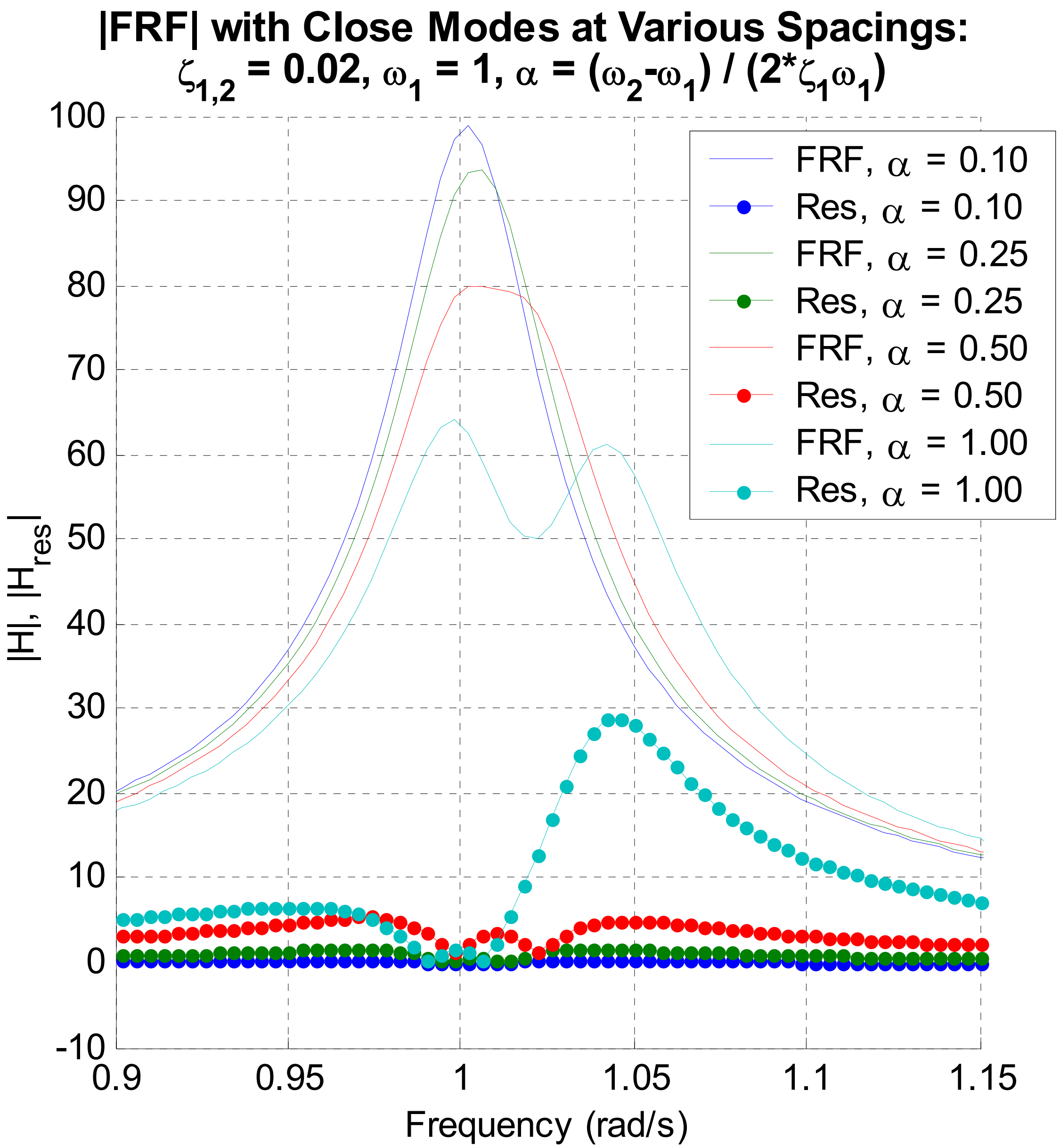


All FRFs

Composite FRF

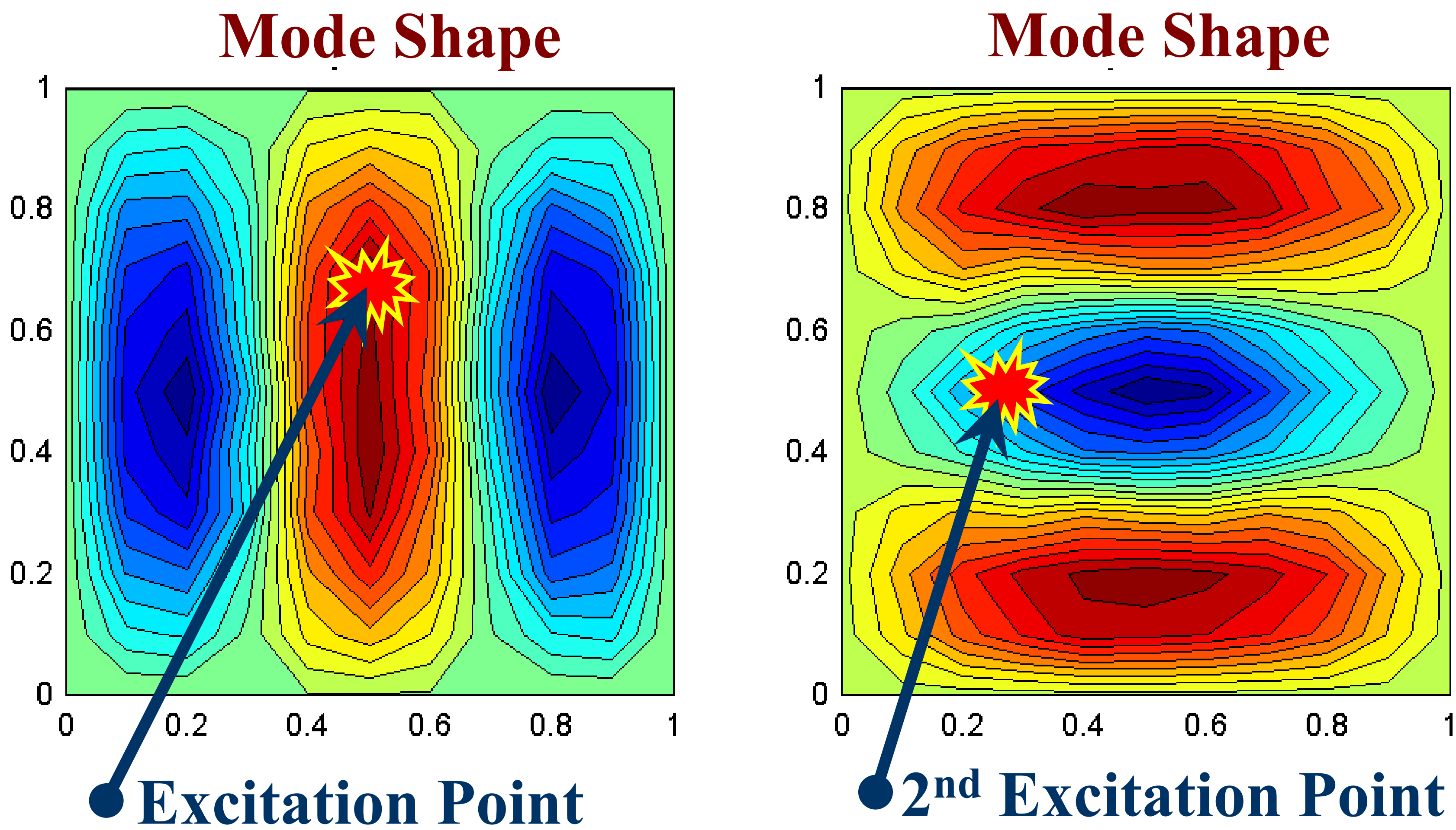
AMI Begins by considering data around the dominant peak.

Modes with Close Natural Frequencies



- Identifying modes with close natural frequencies:
  - Two modes look like a single mode if their natural frequencies are sufficiently close.
  - A MIMO approach is needed!

Repeated Natural Frequencies



•Only a MIMO Experiment can reliably identify modes with repeated natural frequencies.

Mode Indicator Functions (MIFs)

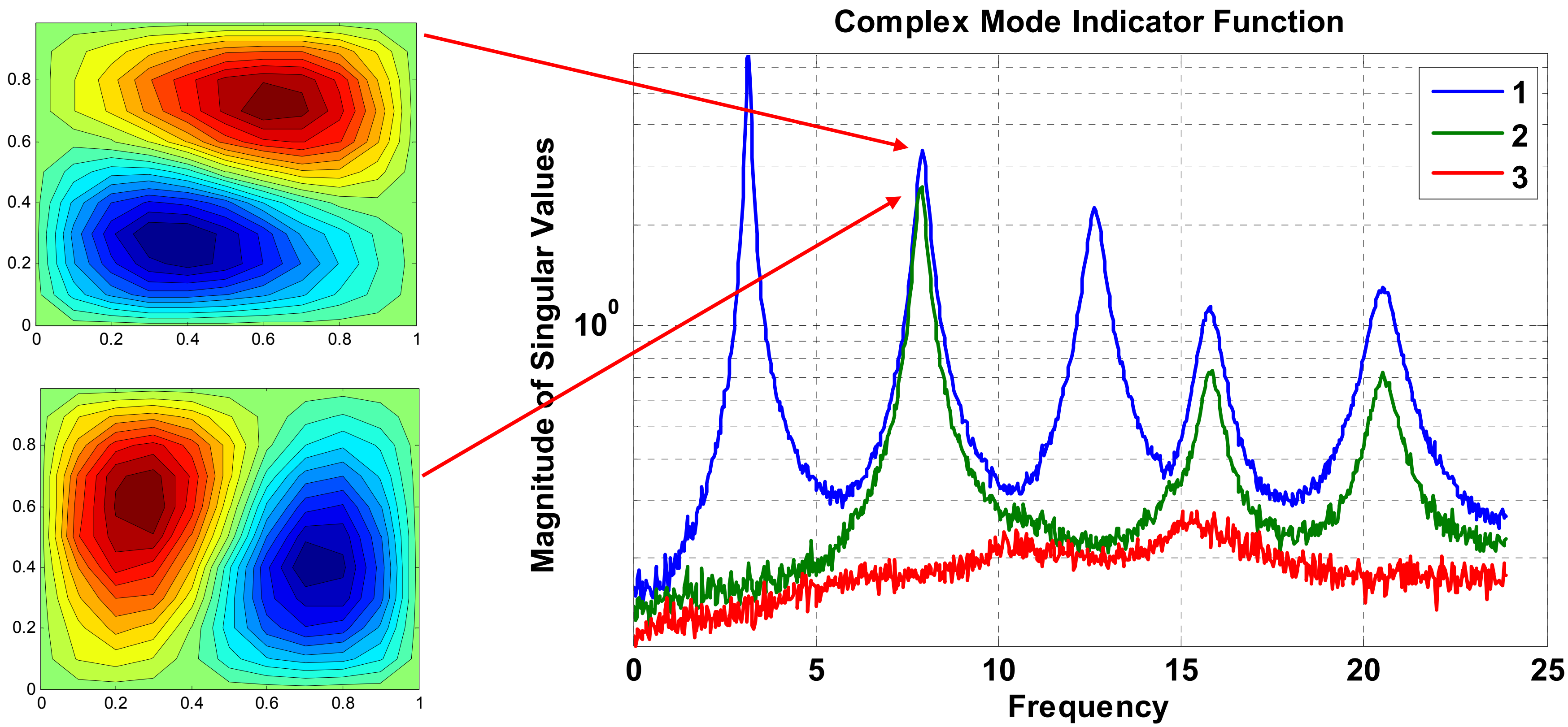
Summary of six mode indicator functions

MIF	Minimisation (maximisation) problem	Rayleigh quotient	Equivalent eigenvalue formulation	Reference
RMIF	$\min \lVert x_R - \lambda x_I \rVert^2$	$\min \frac{\{f\}^T[HI]^T[HR]\{f\}}{\{f\}^T[HI]^T[HI]\{f\}} = \lambda$	$[HI]^+[HR]\{f\} = \lambda\{f\}$	[8] [10]
MMIF	$\min \frac{\lVert x_R \rVert^2}{\lVert x_R + i x_I \rVert^2} = \alpha$	$\min \frac{\{f\}^T[A]\{f\}}{\{f\}^T([A]+[B])\{f\}} = \alpha$	$[A]\{f\} = \alpha([A]+[B])\{f\}$	[12]
MRMIF	$\min \frac{\lVert x_R \rVert^2}{\lVert x_I \rVert^2} = \mu$	$\min \frac{\{f\}^T[A]\{f\}}{\{f\}^T[B]\{f\}} = \mu$	$[A]\{f\} = \mu[B]\{f\}$	[15]
CoMIF	$\min \frac{\{x_R\}^T\{x_R\}}{\{f\}^T\{f\}} = \gamma$	$\min \frac{\{f\}^T[A]\{f\}}{\{f\}^T\{f\}} = \gamma$	$[A]\{f\} = \gamma\{f\}$	[3] [18, 21]
ImMIF	$\max \frac{\{x_I\}^T\{x_I\}}{\{f\}^T\{f\}} = \delta$	$\min \frac{\{f\}^T[B]\{f\}}{\{f\}^T\{f\}} = \delta$	$[B]\{f\} = \delta\{f\}$	[6] [22]
CMIF			$[H]^H[H]\{f\} = \sigma\{f\}$	[24, 25]

Notation: $[A] = [HR]^T[HR]$, $[B] = [HI]^T[HI]$, $[HI]^+ = ([HI]^T[HI])^{-1}[HI]^T$.

- M. Rades, “Comparison of some Mode Indicator Functions,” MSSP, 1994, 8(4), 459-474.

Sample CMIF: Plate



- CMIF for 81 output, 3 input FRF matrix for Plate shows two dominant singular values at several peaks, indicating the presence of two distinct shapes.
- Contour plots show the shapes of the two dominant singular vectors near the first peak.

MAC and MSF

- Modal Assurance Criterion or MAC evaluates the linear independence of two vectors (in a Euclidean Space):

$$MAC_{cdr} = \frac{\left| \{\psi_{cr}\}^H \{\psi_{dr}\} \right|^2}{\{\psi_{cr}\}^H \{\psi_{cr}\} \{\psi_{dr}\}^H \{\psi_{dr}\}} = \frac{\left(\{\psi_{cr}\}^H \{\psi_{dr}\} \right) \left(\{\psi_{dr}\}^H \{\psi_{cr}\} \right)}{\{\psi_{cr}\}^H \{\psi_{cr}\} \{\psi_{dr}\}^H \{\psi_{dr}\}}$$

- Modal Scale Factor (MSF) compares the scale of two vectors that are typically already known to have similar shapes.

$$MSF_{cdr} = \frac{\{\psi_{cr}\}^H \{\psi_{dr}\}}{\{\psi_{dr}\}^H \{\psi_{dr}\}}$$

Hybrid, MIMO-AMI

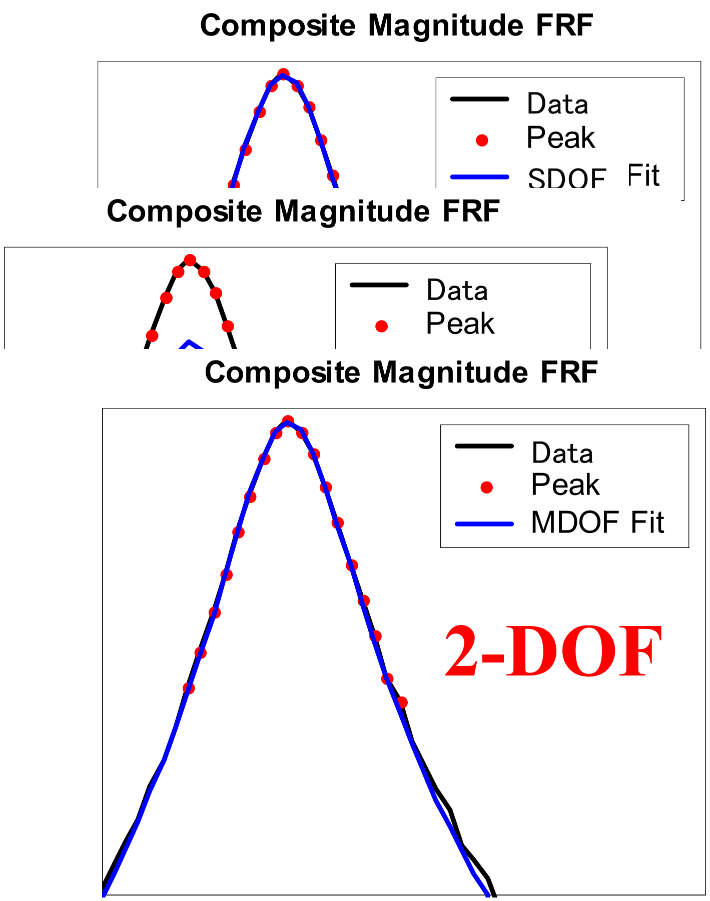
FRFs → Try SDOF Common Denominator →

✓ Check Agreement

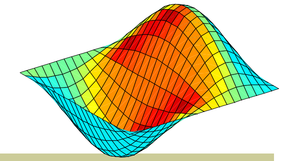
✓ Check Singular Value Metric (SR)

Try MDOF

Retain & Subtract



Simply Supported Plate



- Simply Supported Rectangular Plate:
  - Aspect ratio = 1.001
    - Pairs of modes separated by 2%, 3%, 4% of their half power bandwidths.
    - Gaussian noise added to impulse responses.
    - 25 modes used to construct impulse response.
    - 8 modes in frequency band of interest.
  - 81 Outputs, 3 Inputs **=>243 FRFS**

Representative Magnitude FRF

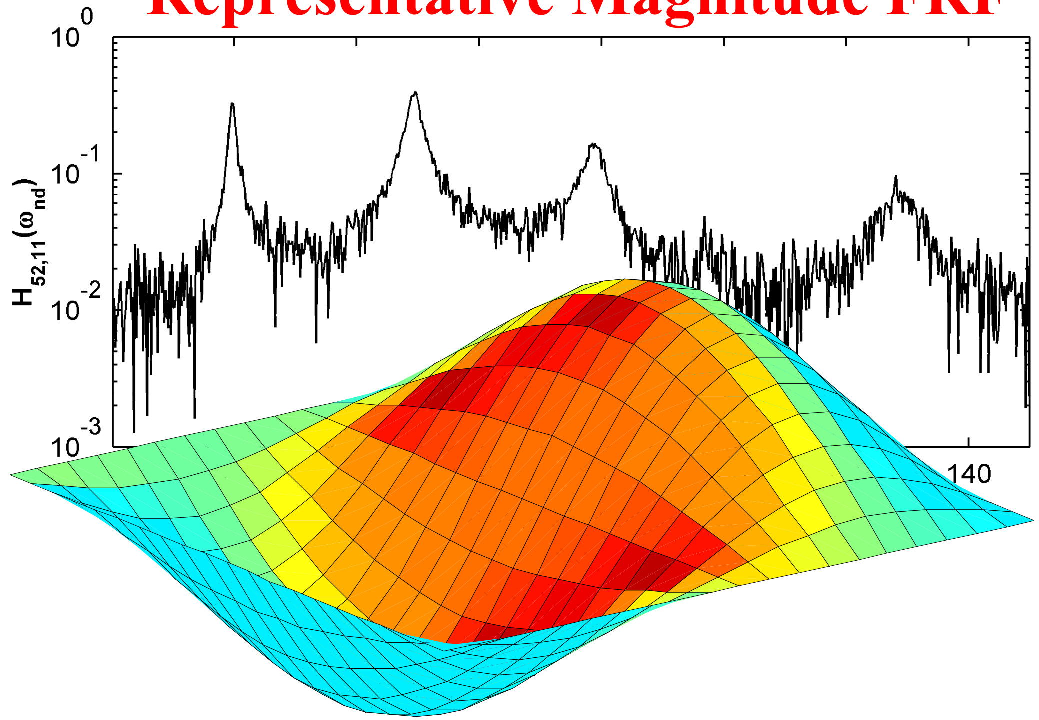
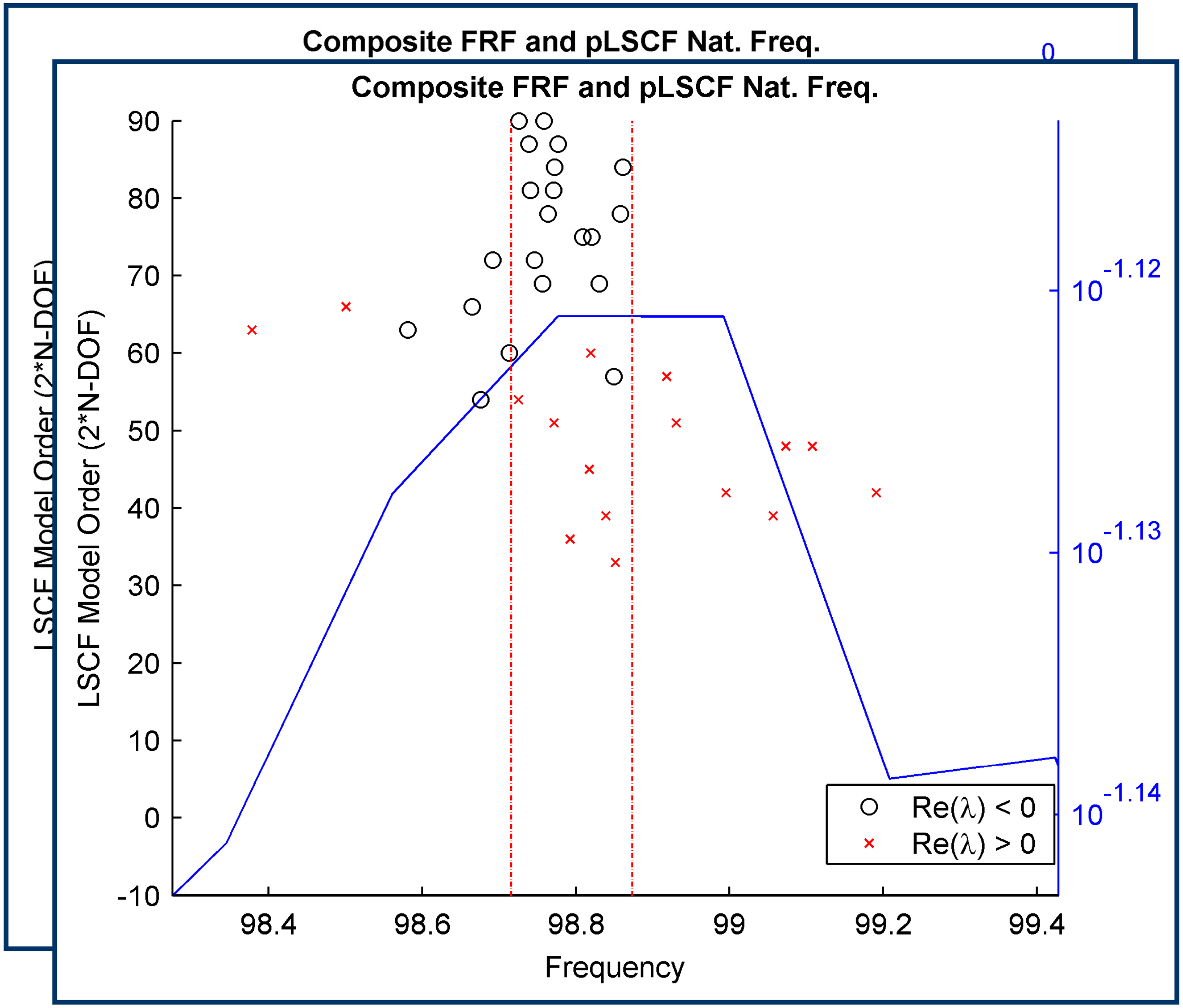


Plate Data: pLSCF Algorithm



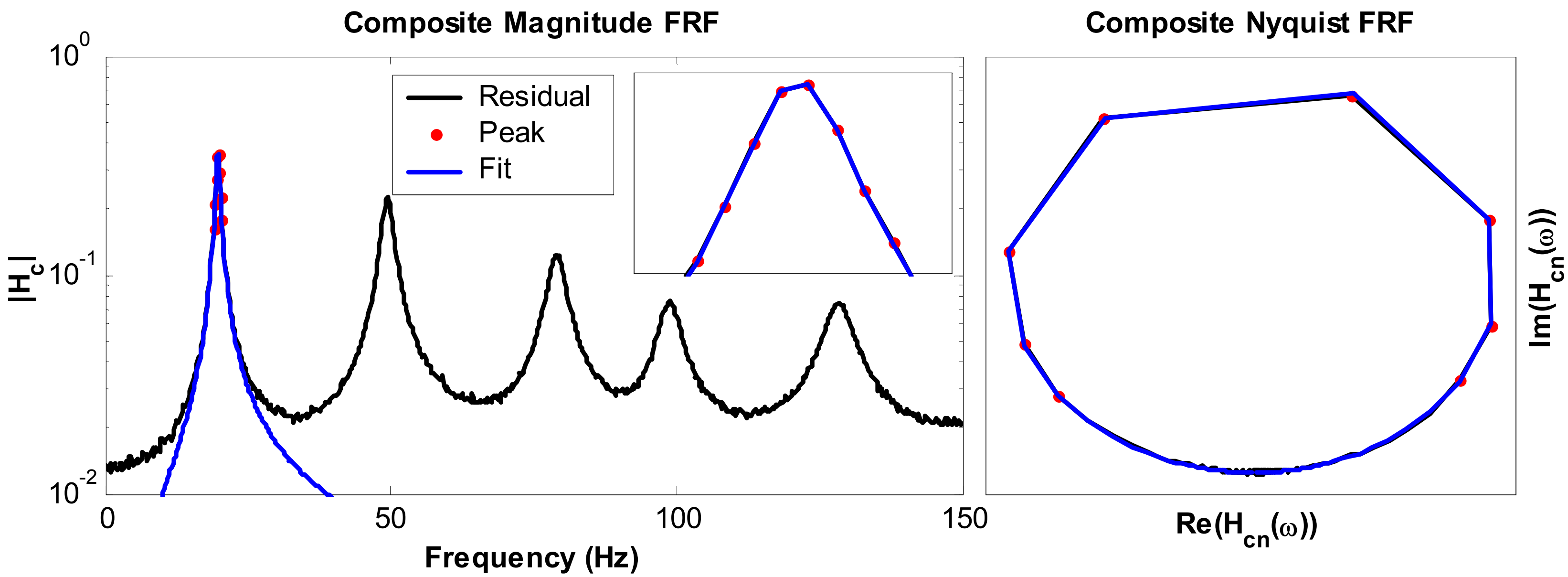
pLSCF

- [Guillaume *et al* 2003]
- pLSCF generally outperforms other frequency domain algorithms.

For this Problem:

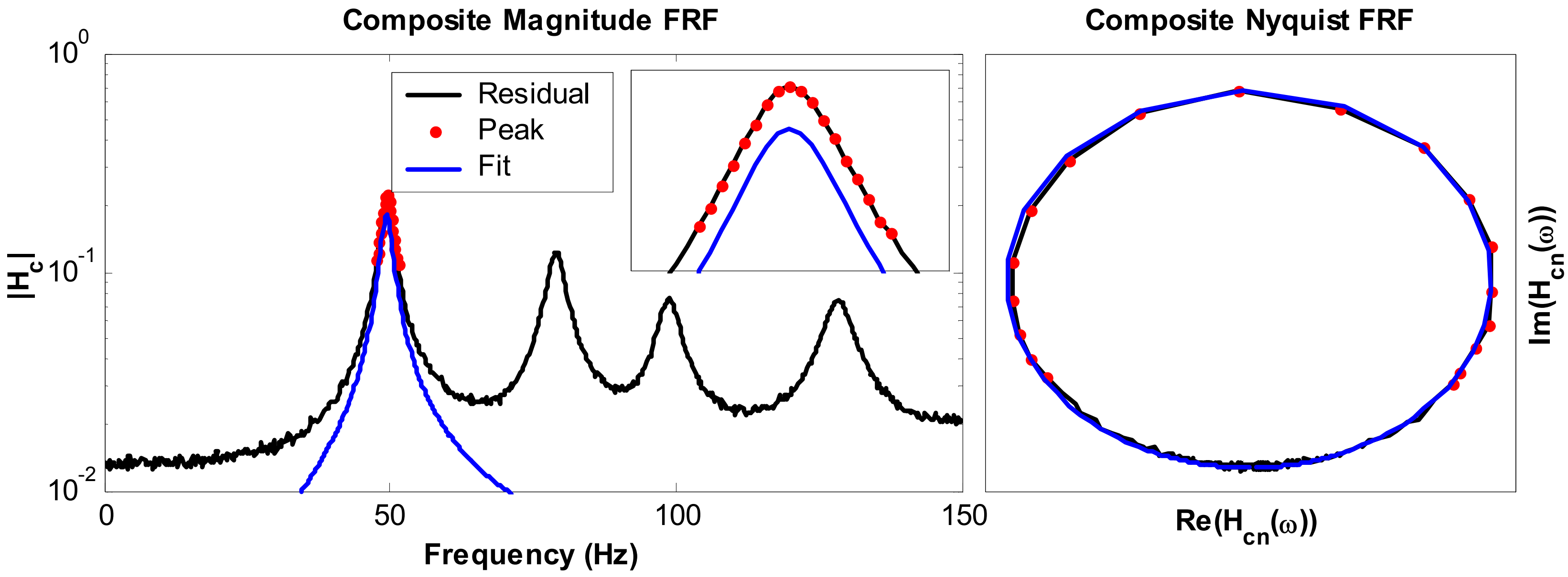
- Clear stabilization diagram when unstable modes are ignored.
- Distinction between close modes is blurred at 2 of 3 peaks.

Analysis with AMI (1)



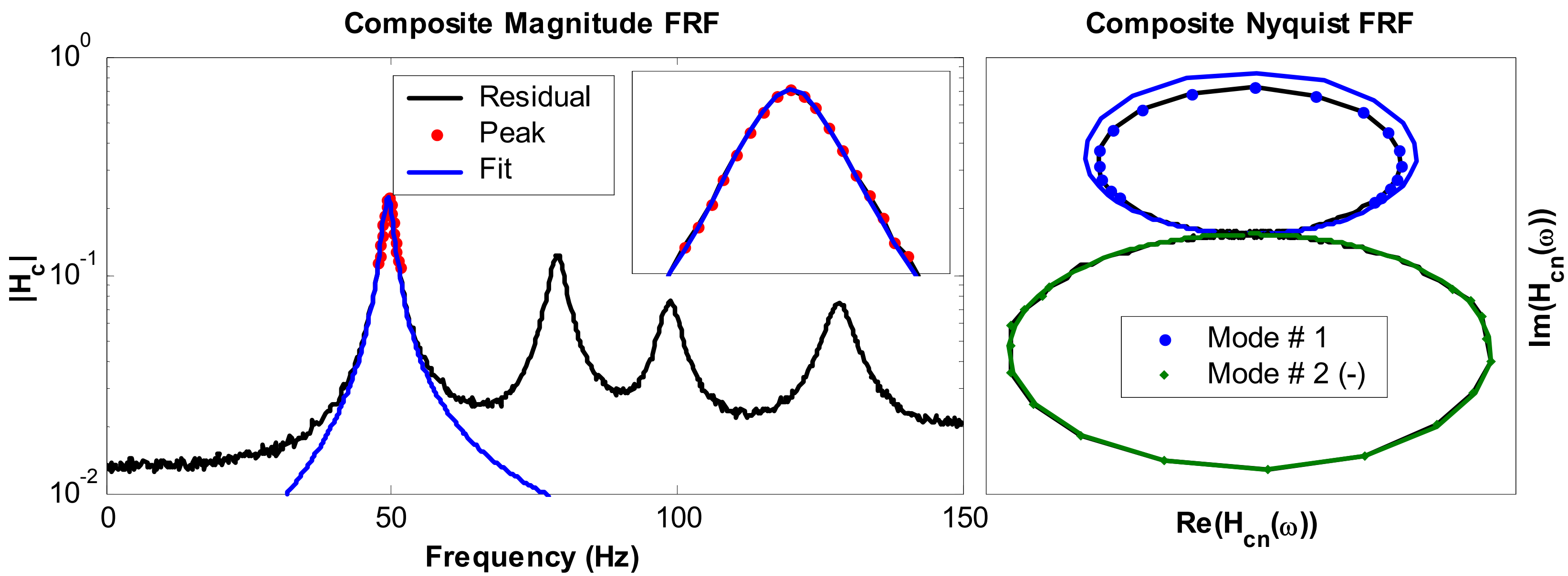
- First subtraction step identifies the first mode.
- Singular value metric indicates that one mode is present. ***SR ~ [1.0,0.1,0.1]***

Analysis with AMI (2)



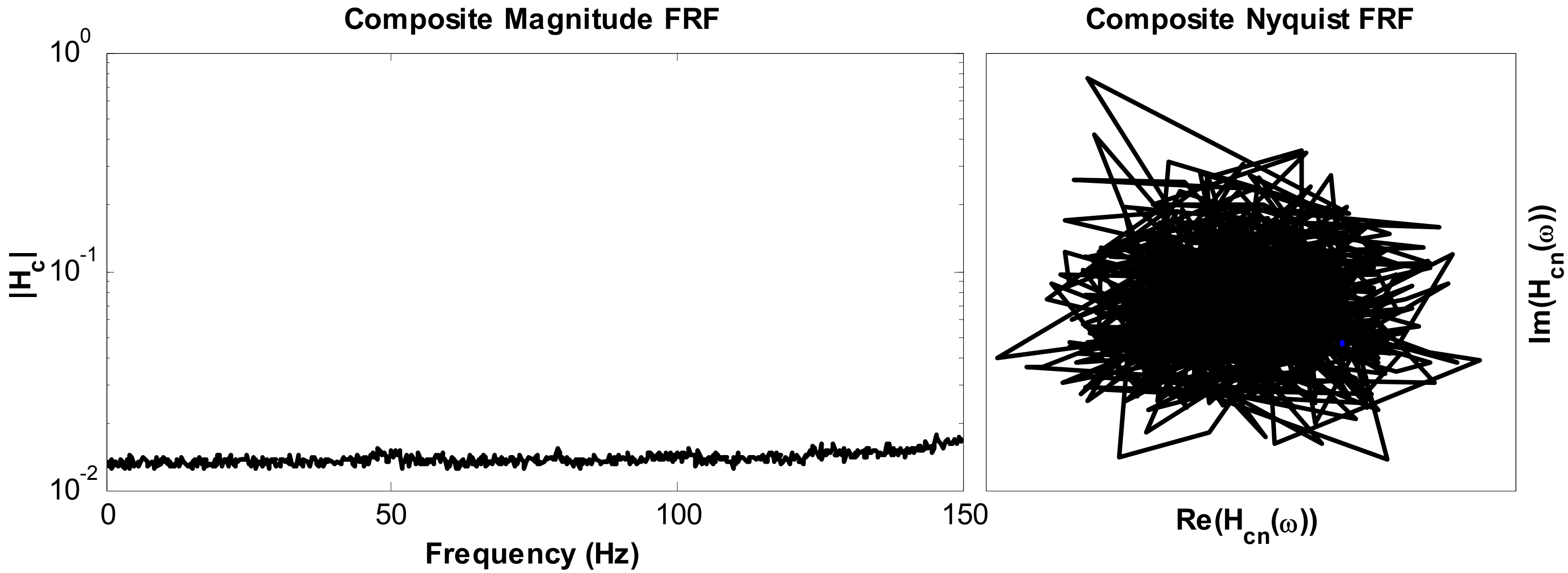
- Second Subtraction Step – SDOF fit.
- Singular value metric suggests that two modes are present. ***SR ~ [1.0,0.5,0.1]***
- Reconstruction uniformly underestimates the peak.

MIMO-AMI Subtraction(3)



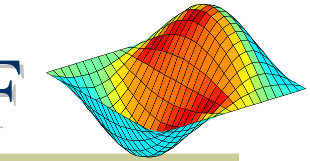
- Second Subtraction Step – MDOF fit.
- A two-mode fit agrees well with the data.
- The rest of the peaks were processed in a similar manner.
- Automatic processing was possible for this data set.

Analysis with AMI (4)



- After five subtraction steps the data has been reduced to noise.
- Eight modes have been identified

Plate Results: AMI and pLSCF

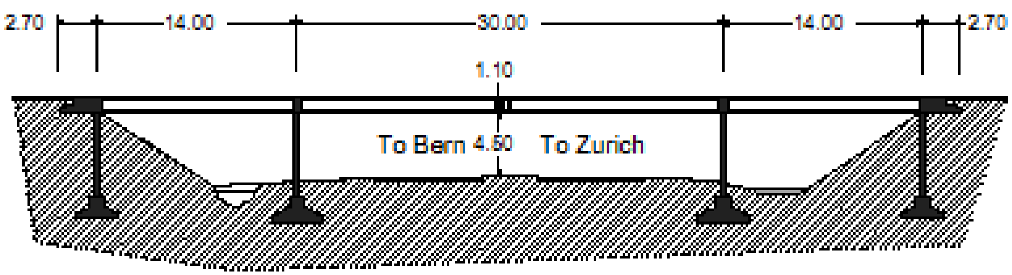


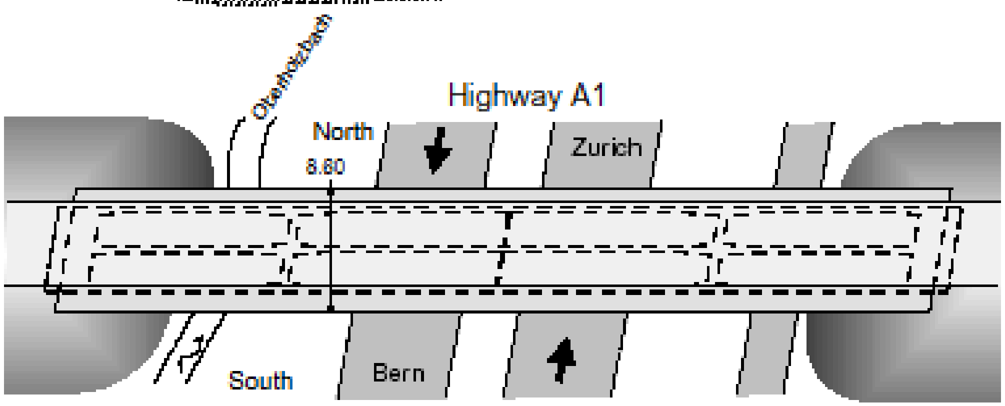
	Actual		MIMO-AMI		pLSCF*	
Mode	ω_r	ζ_r	ω_r	ζ_r	ω_r	ζ_r
1	19.759	0.02	19.758	0.0199	19.756 - 19.762	0.0185 - 0.0189
2	49.3678	0.02	49.369	0.0202	49.349 - 49.373	0.0171 - 0.0183
3	49.427	0.02	49.421	0.0200	49.417 - 49.425	0.0185 - 0.0191
4	79.0358	0.02	79.029	0.0194	79.015 - 79.035	0.0172 - 0.0187
5	98.716	0.02	98.705	0.0191	98.692 - 98.809	0.00329 - 0.0160
6	98.874	0.02	98.855	0.0194	98.758 - 98.861	0.000426 - 0.0128
7	128.384	0.02	128.366	0.0195	128.22 - 128.52	0.00171 - 0.0171
8	128.483	0.02	128.508	0.0196	128.38 - 129.66	0.00279 - 0.0170

*pLSCF: Range of values for model orders 69-90.

- The pLSCF results can have much larger errors, depending on the model order chosen.
- pLSCF underestimates damping.
- AMI distinguishes the pairs of close modes.

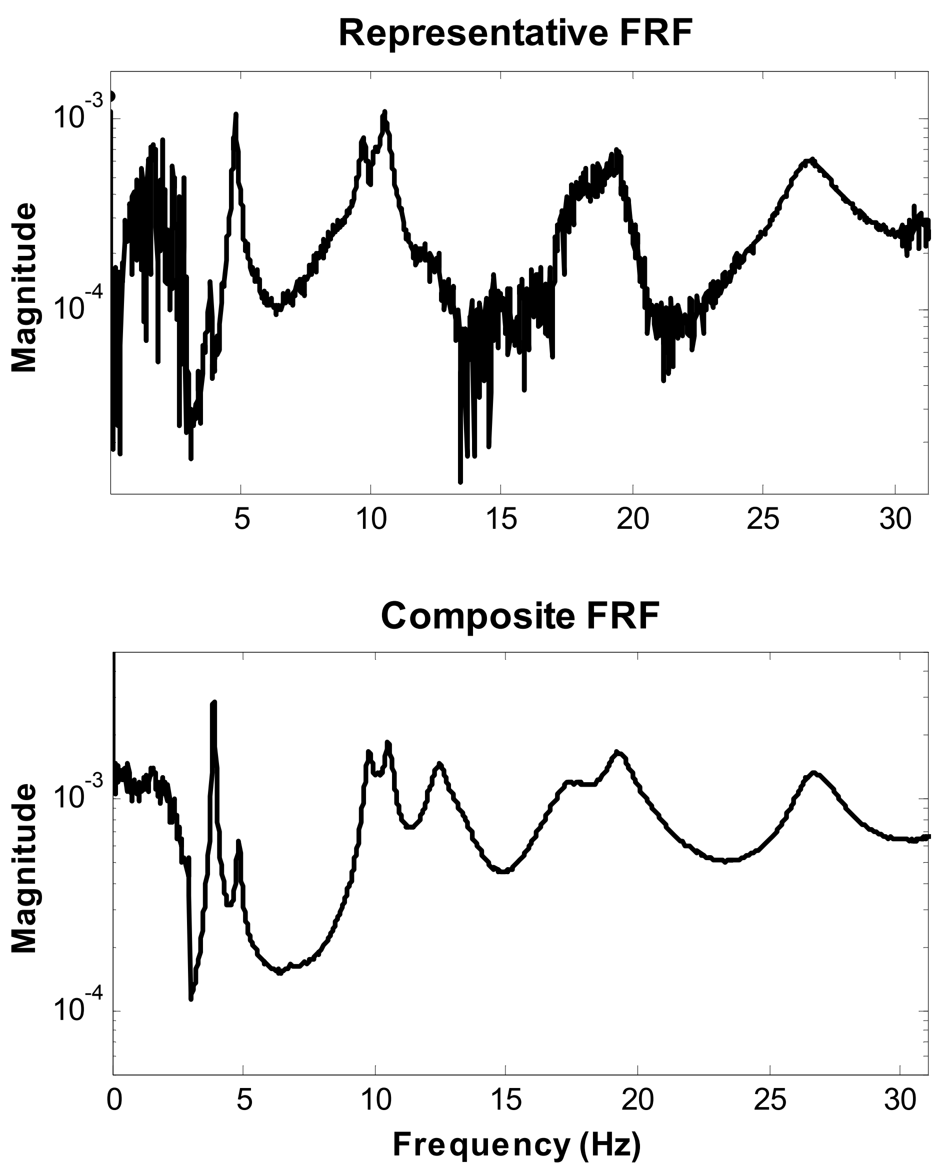
Experimental Application: Z24 Bridge





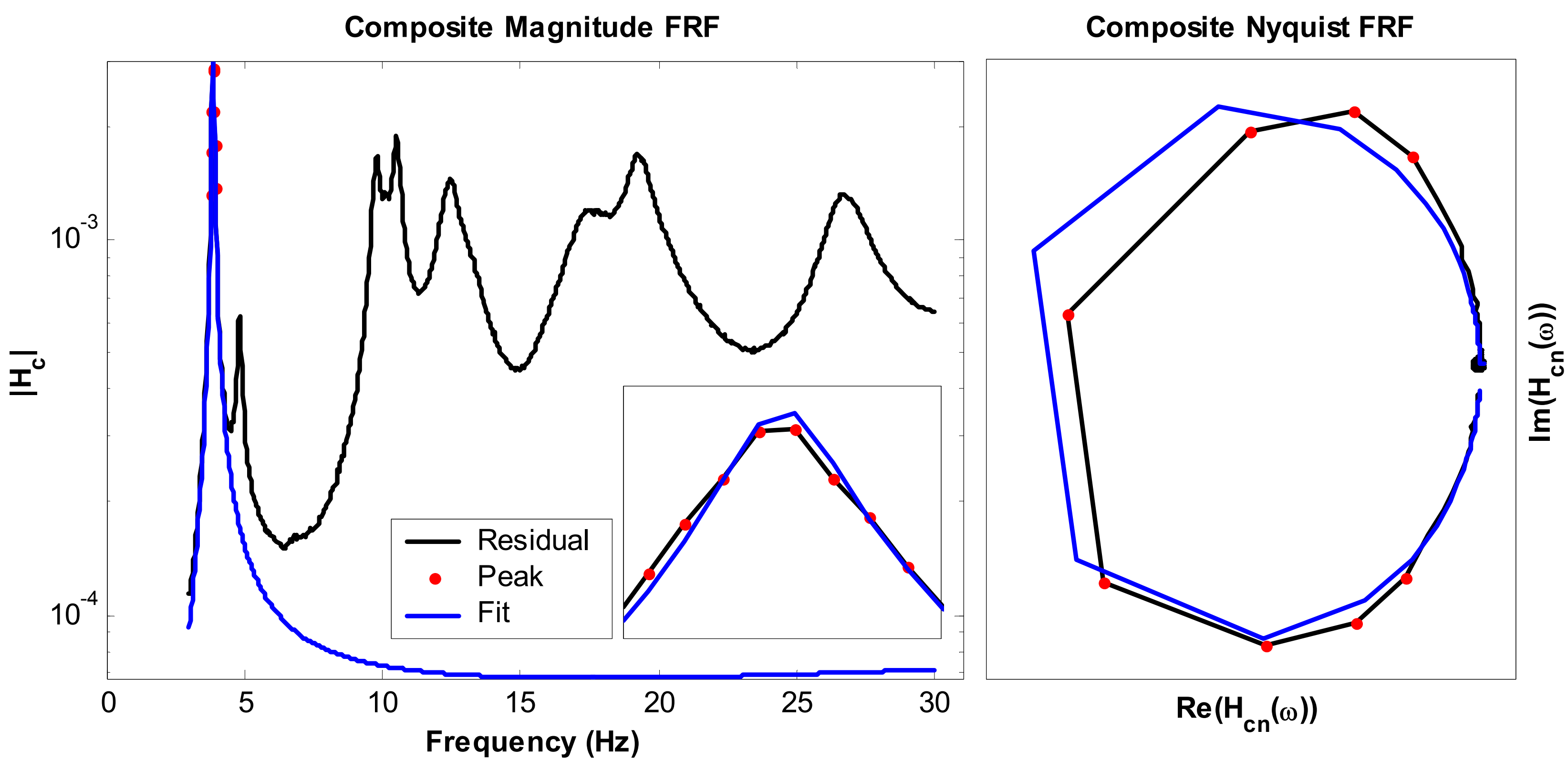
- AMI was also applied to data from the Z24 highway bridge in Switzerland.
- Data courtesy of the Catholic University of Leuven (KUL) in Belgium.
  - Part of a Progressive Damage – Condition Monitoring study.
  - Excited by 2 shakers.
  - Response measured at 297 points.
  - KUL Provided data to interested researchers.

Z24 FRF Data



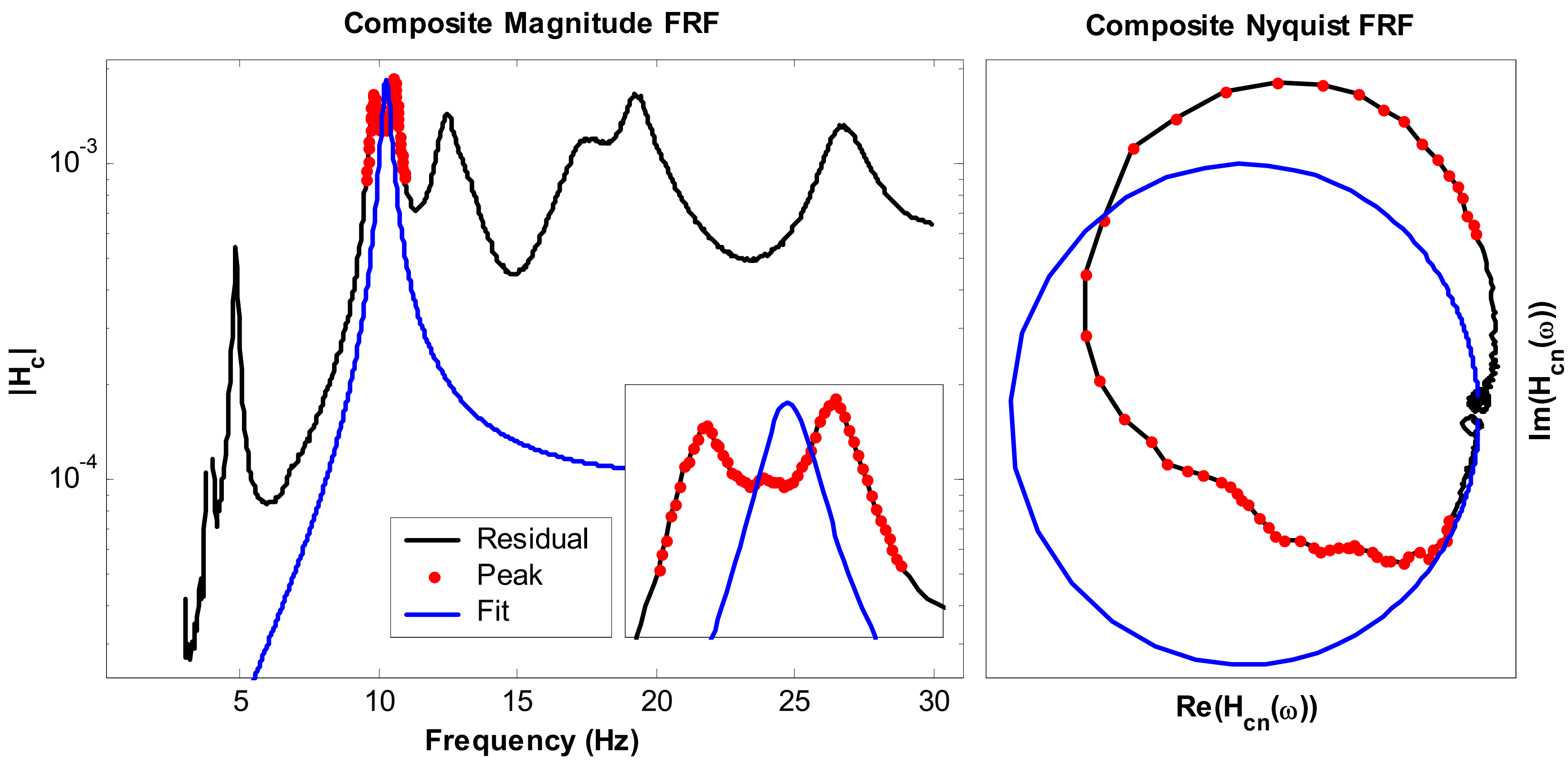
- 150 FRFs created from the time data provided by KUL.
  - 2 inputs
  - 75 outputs
  - H_1 method used to find FRF matrix.

Z24 Bridge – AMI Subtraction (1)



- Agreement is imperfect for the first (dominant) mode.
- Singular value metric indicates that only one mode is present.

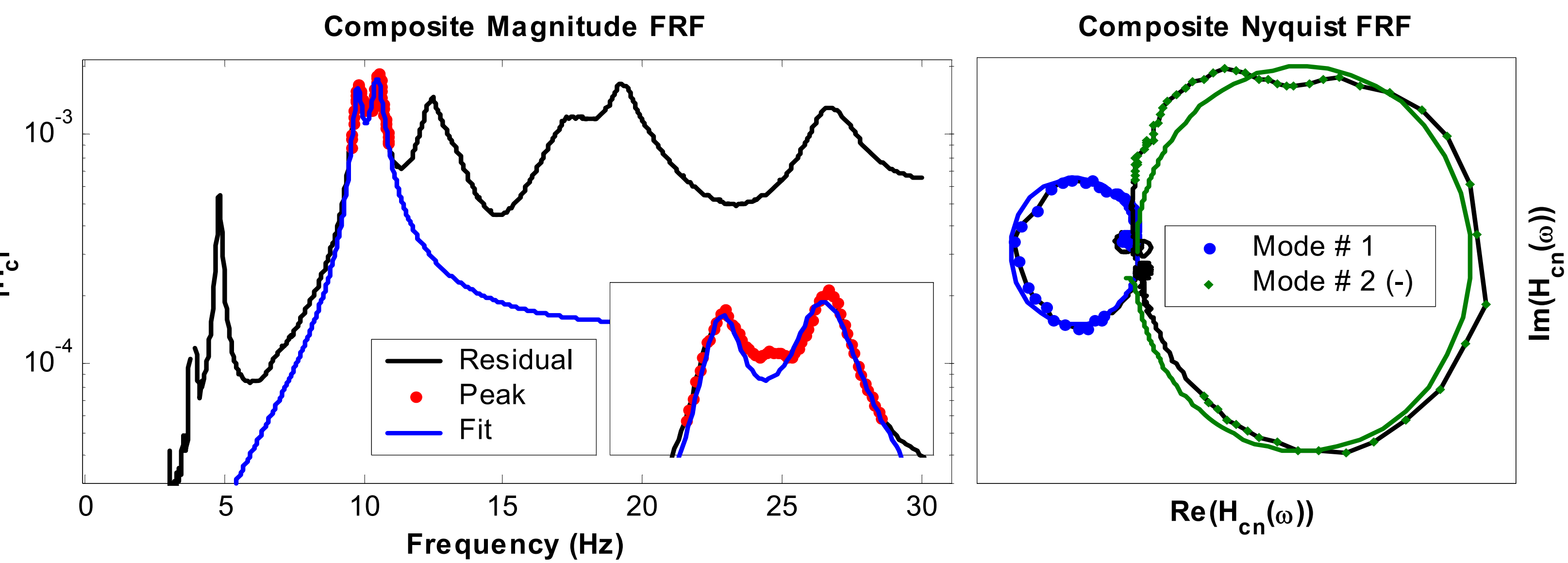
Z24 Bridge – AMI Subtraction (2)



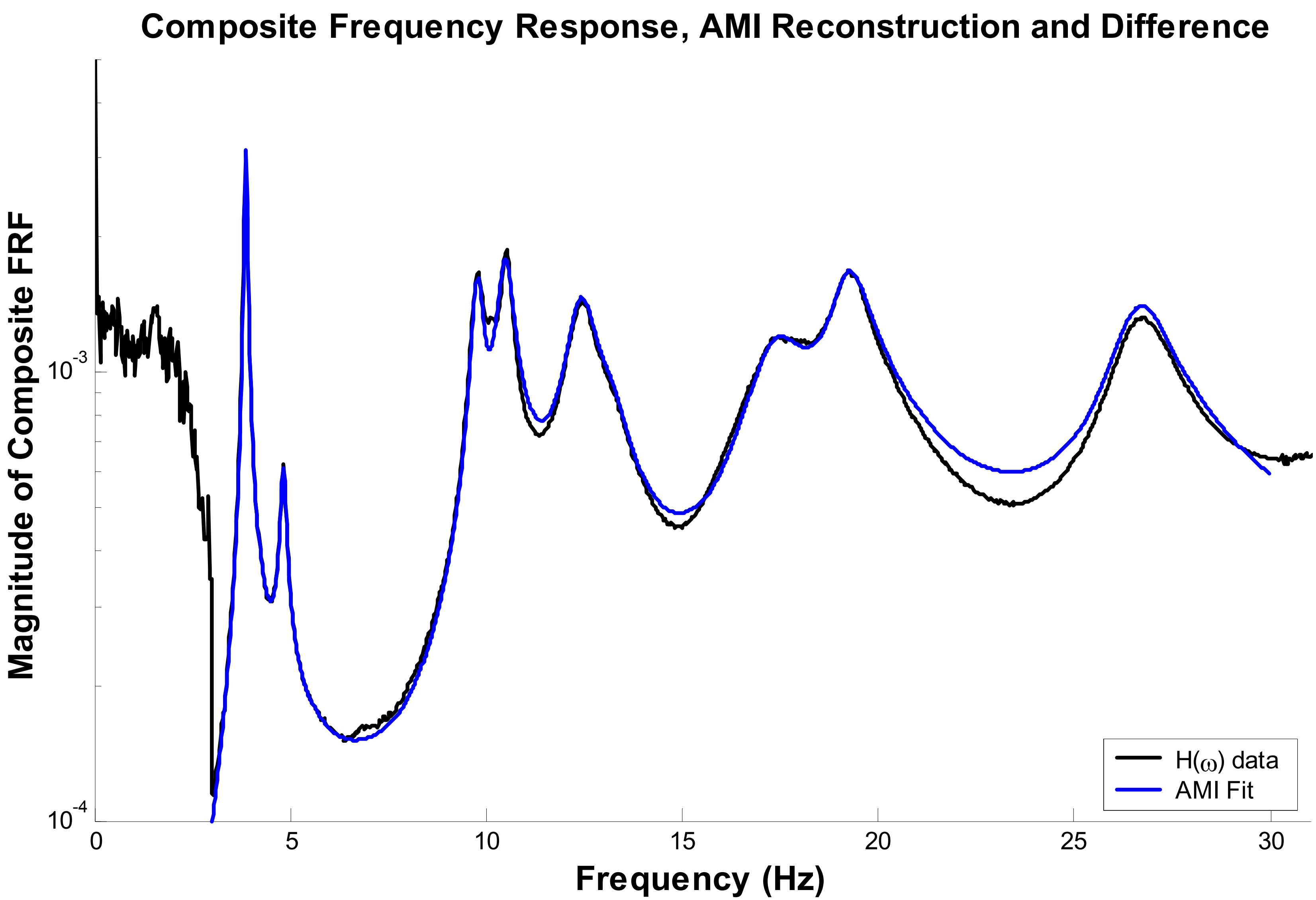
- Two modes are clearly evident at the second peak.
- This could have been treated as two peaks.

Z24 Bridge – AMI Subtraction (3)

2 - Modes

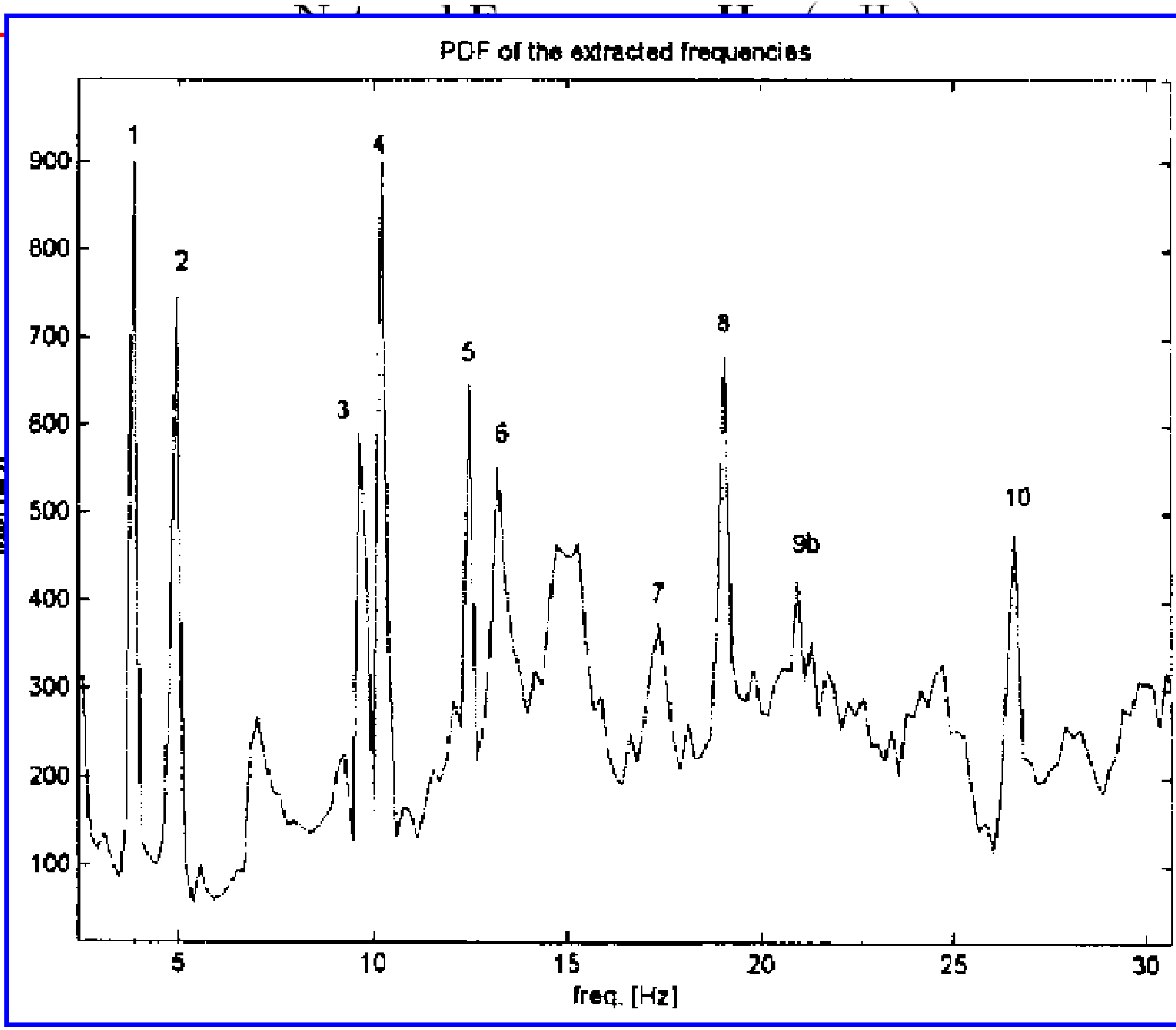


Z24 – After Mode Isolation



Natural Frequencies

Mode	...z & ...on [60]
1	...8
2	...3
3	...7
4	...7
5	...1
6	
7	...2
8	...4
9	
10	...8
11	



- A ... sa ... K (2 ... p

Mode Shapes

- The mode shapes found by AMI agree well with those presented by other researchers.

Mode 1: 3.9 Hz

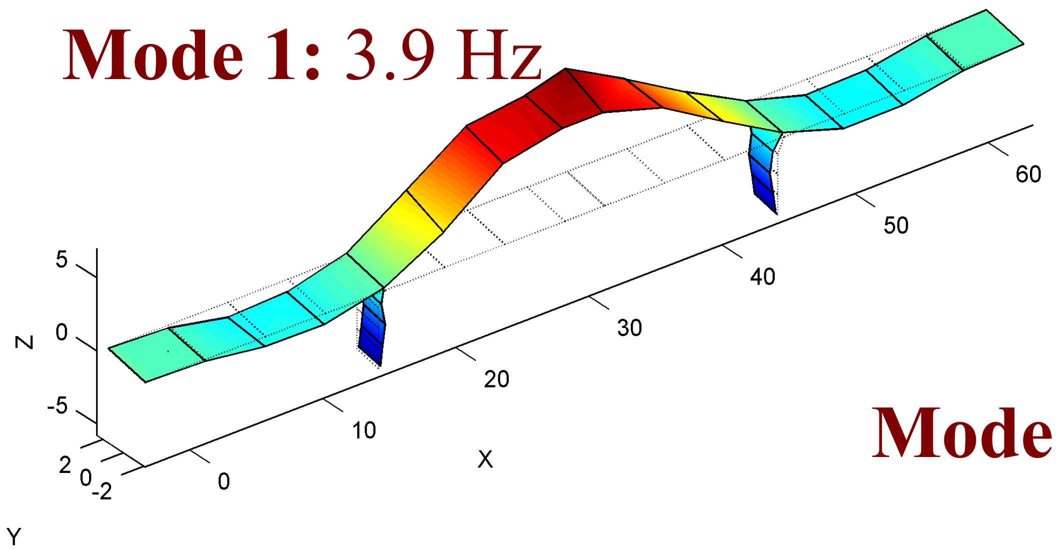
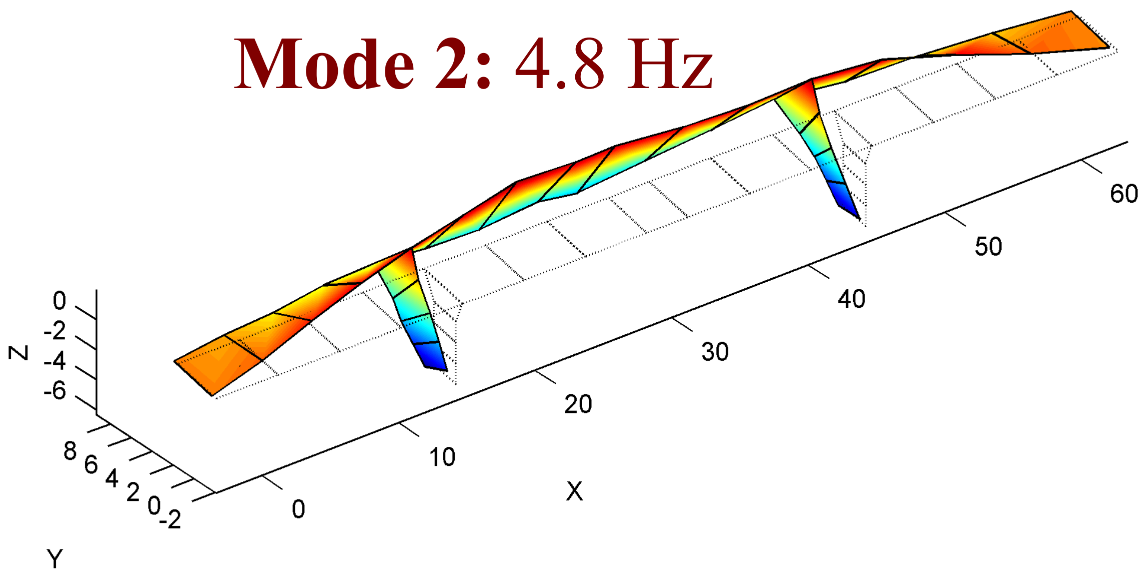
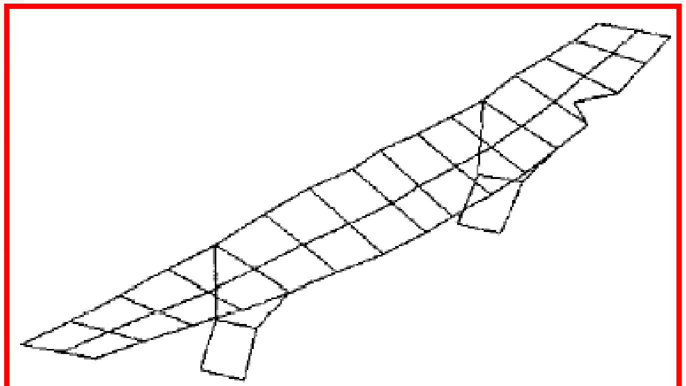


Figure 5: Mode 1 (Ambient)

Marchesiello et al 2001

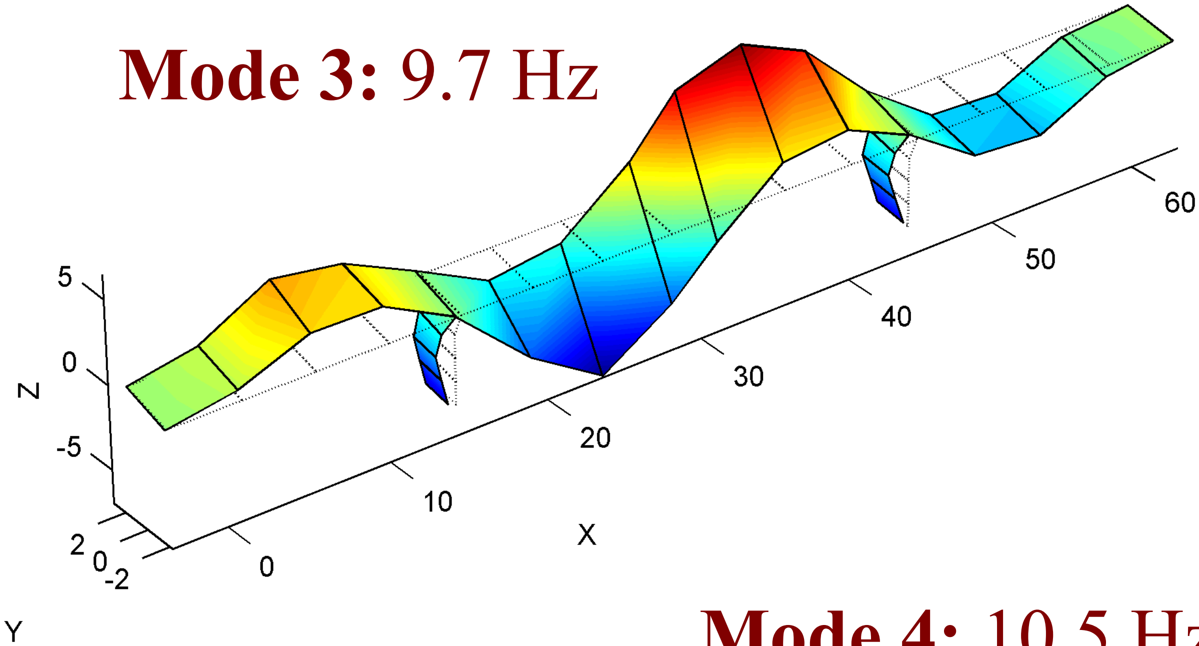
Mode 2: 4.8 Hz



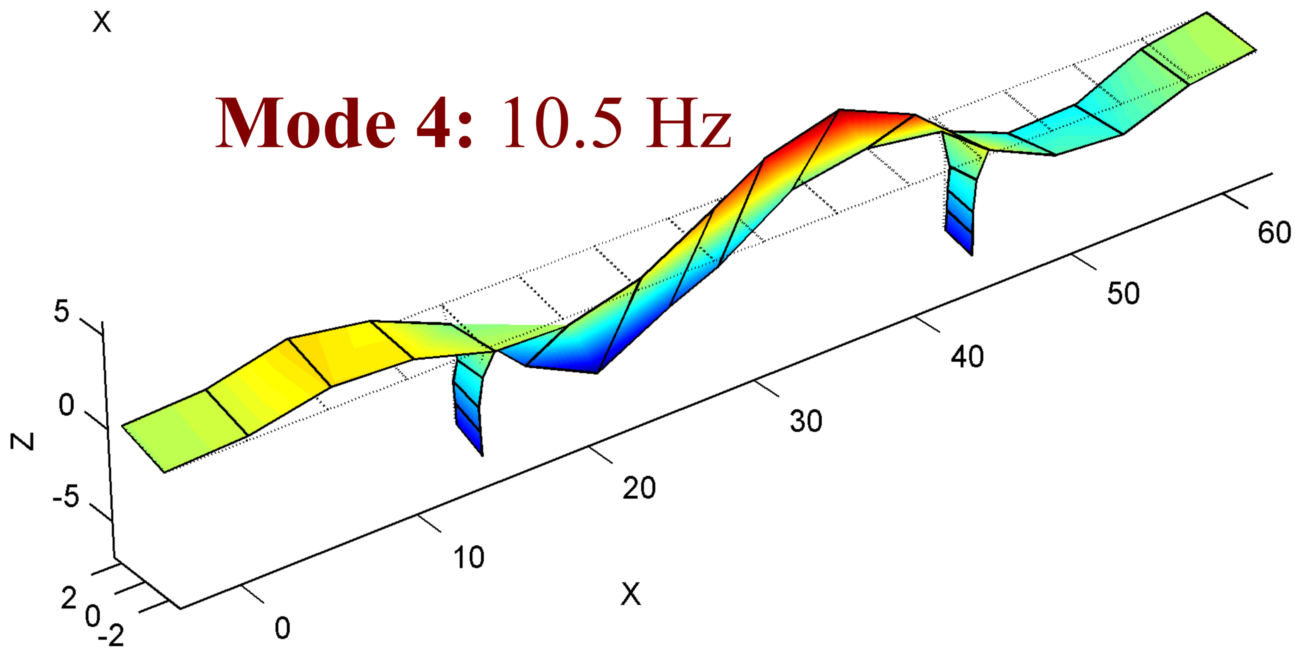


Mode Shapes (2)

Mode 3: 9.7 Hz

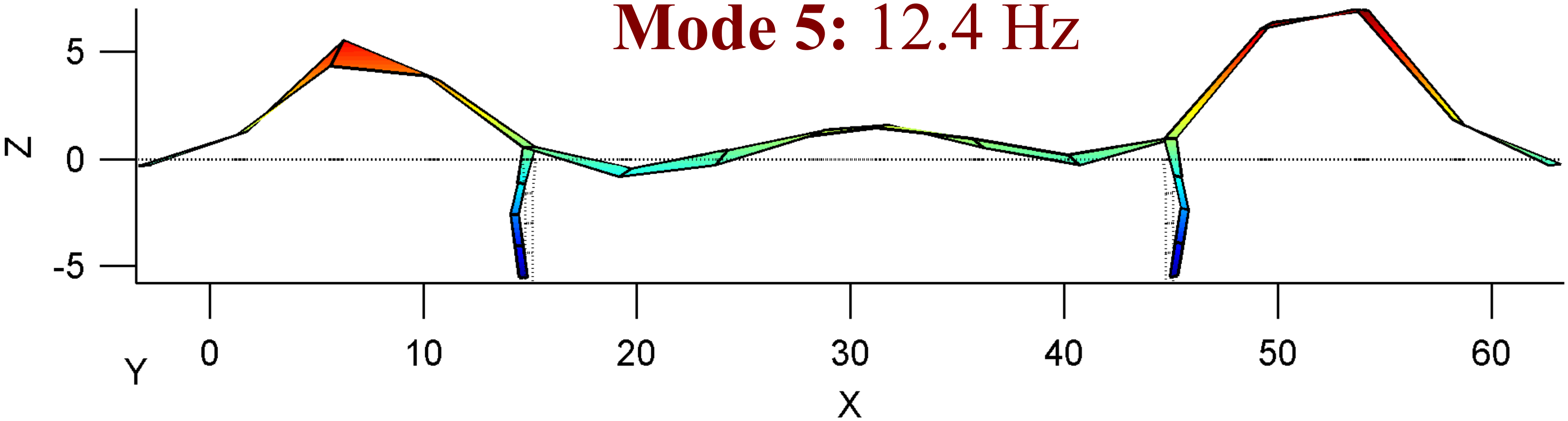


Mode 4: 10.5 Hz

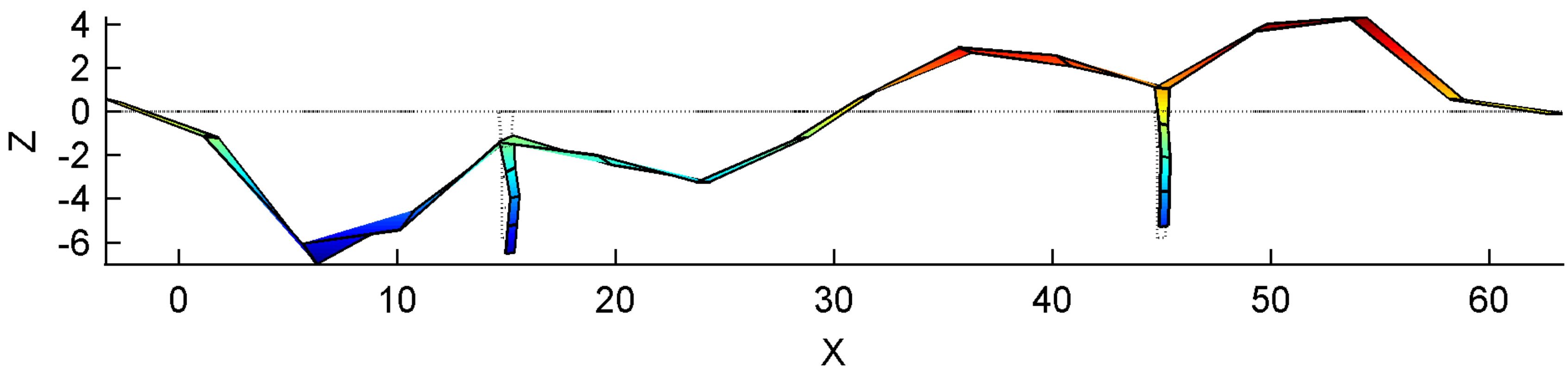


Mode Shapes (3)

Mode 5: 12.4 Hz

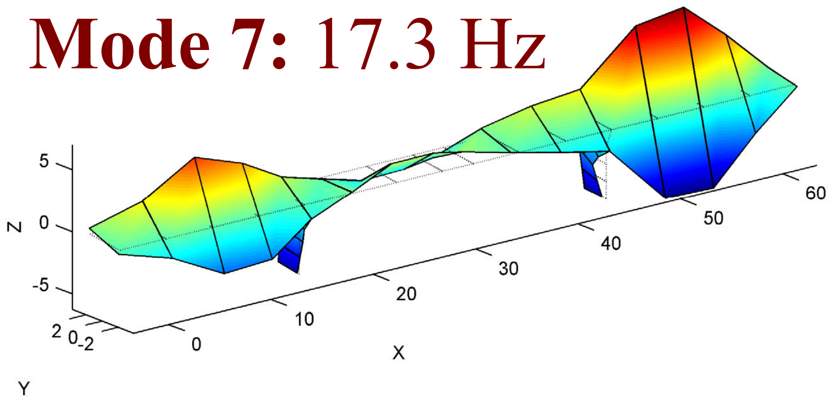


Mode 6: 13.2 Hz

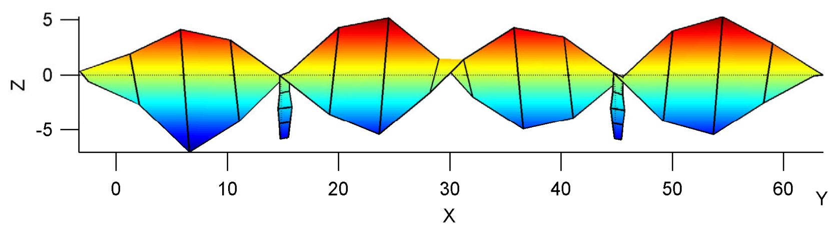


Mode Shapes (4)

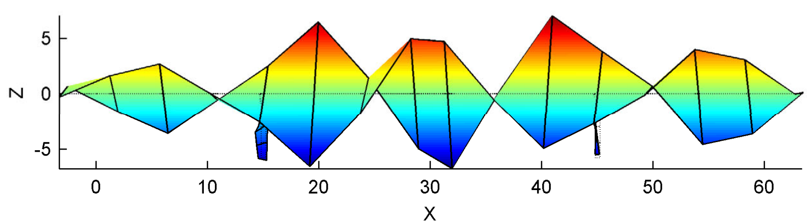
Mode 7: 17.3 Hz



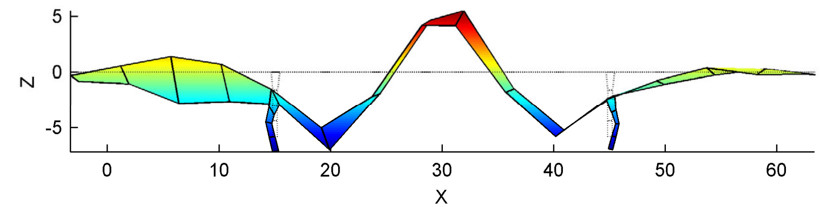
Mode 8: 19.3 Hz



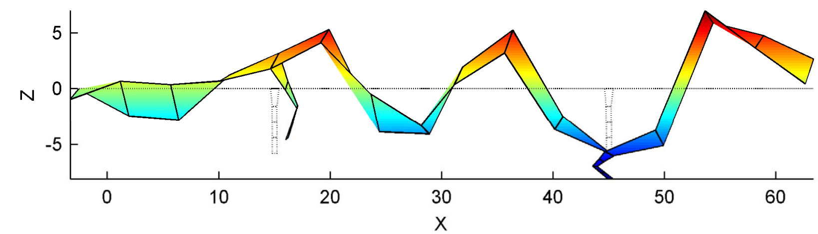
Mode 10: 26.7 Hz



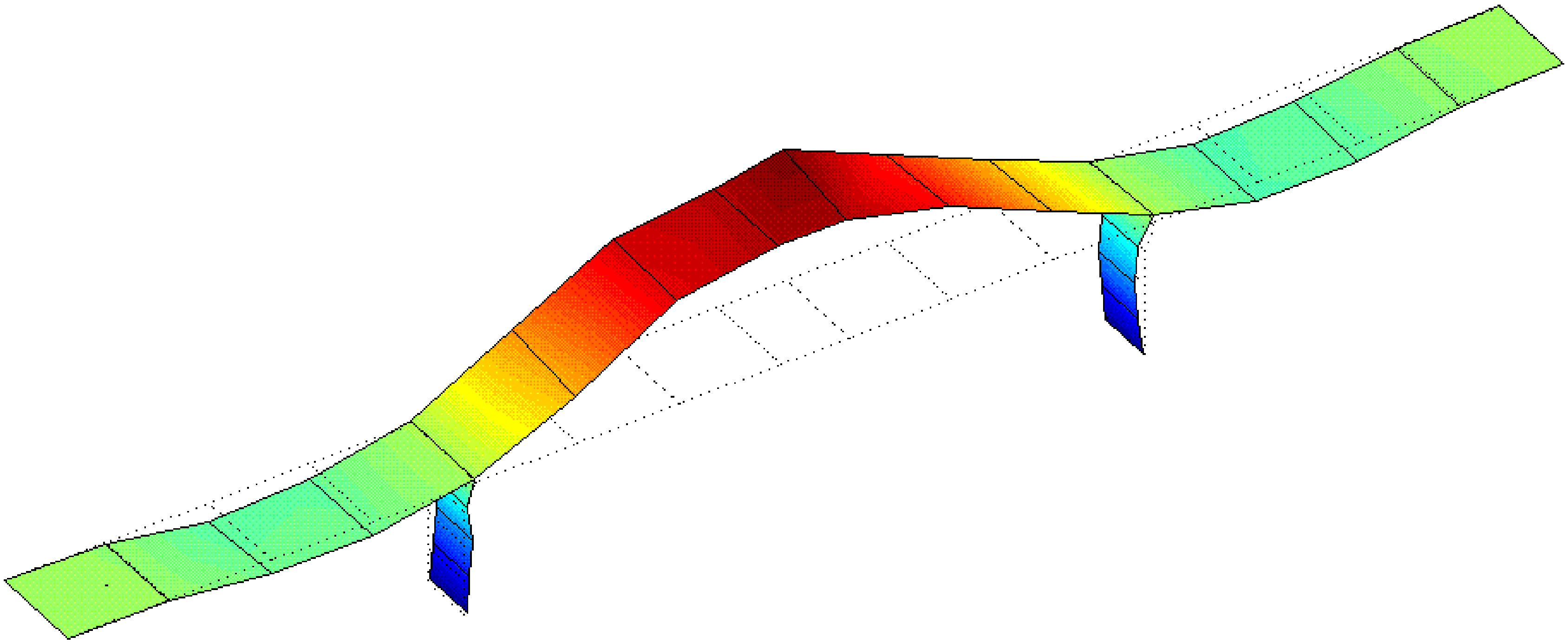
Mode 9: 19.5 Hz



Mode 11: 28.1 Hz

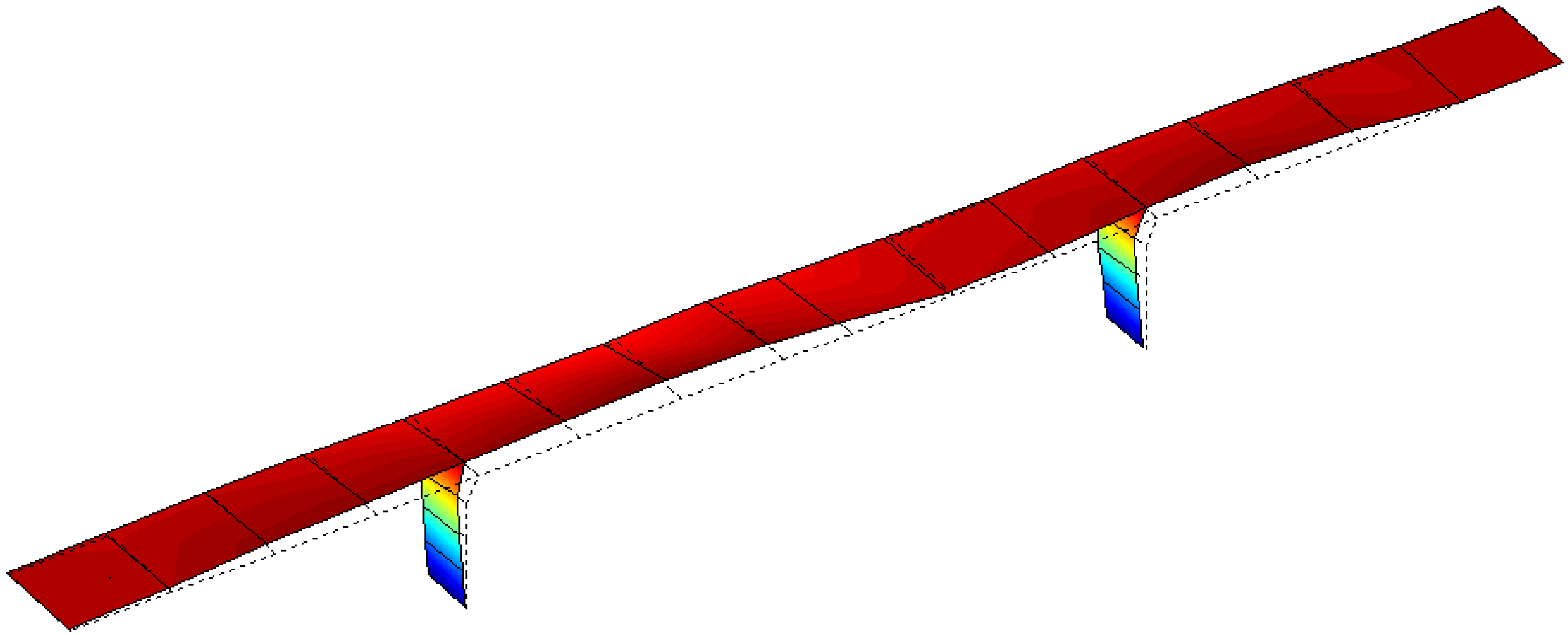


Mode Shape Animations: 1st Mode



- The low-frequency modes of the bridge had high Modal Phase Co-linearity
  - i.e. all points achieve their maxima at the same time.
  - Typical of undamped structures. (real modes)

Mode Shape Animations: 2^{nd} Mode



Appendix

- The following slides contain detailed instructions showing how to run the AMI algorithm to extract modes from FRF measurements.

How to Run AMI in MATLAB

- AMI is a basic function that is run from the command line in MATLAB, operating on an array of FRF measurements (here denoted **H**) and a corresponding frequency vector (here denoted **ws**).
  - The FRF data, H, is an array of size Nf × No × Ni
    - Nf = number of frequency lines (length of **ws**)
    - No = number of outputs
    - Ni = number of inputs (if No=1 and Ni>1, transpose the data to create a MISO set)
- Typically a script is created to load the measurements, run AMI and then post process the results. See **runAMI.m** for an example.
  - The data stored in *ffbeam_17_Hjp.mat* is loaded
  - The default options for AMI are loaded into a structure called **AMI_set** using `AMI_set=AMIG_def_opts;`
    - Then individual settings can be modified if needed. Typically one need only set the following:
    - `AMISET.DVA = 'A'; % this tells AMI that H is an acceleration/force FRF`
    - `AMISET.AutoSubLevel = 0.5; % Tells AMI to subtract modes automatically until the largest peak in the measurements is reduced to 50% of what it was initially.`
- AMI is then called using:
  - `>>ami(H,ws,AMI_set)`

Other AMI Options

**** See AMIG_def_opts.m for full list of options!**

```
% ****************** DATA DESCRIPTION: ***********************************
% DVA = ('D','V','A') Tells AMI whether the data is:
%           Displacement/Force (D)
%           Velocity/Force (V)
%           Acceleration/Force (A)
% OMA = (0 or 1) Switch between standard modal analysis and operational:
%           (false or 0) [default] H = FRF data for standard EMA
%           (true or 1) H = output spectra for operational modal analysis.
%
% ****************** AMI OPTIONS:  CRITICAL & SEMI-CRITICAL ***************
% NumPoints   = Number of points on each side of max(abs(H)) to use for
%               curve fits during subtraction stage.
%               If num_pts < 1 it is interpreted as a level and all points
%               are taken such that abs(H) > level*max(abs(H)).
% NumPointsIsol = Number of points on each side of max(abs(H)) to use for
%               curve fits during the isolation stage. (see NumPoints)
% IsolIter    = Maximum number of iterations allowed in the Mode Isolation
%               & Refinement stage.
% NoisePoints = Approx noise width in transfer functions.  When finding the
%               points around a peak, AMI steps down until this many points
%               beyond the peak are also below the given level.
% FRF_RF      = Minimum acceptable reduction factor: RF=max(H)/max(H-H_fit).
%               AMI Prompts for a higher order fit if this reduction is not
%               achieved (during subtraction).
% FRF_RF_full = Minimum acceptable reduction factor for full rank-residue
%               model.  RF=max(H)/max(H-H_fit) where H_fit has a residue
%               matrix that may have rank>1 (nonphysical).  This is used to
%               help AMI decide whether to try an MDOF fit.
```

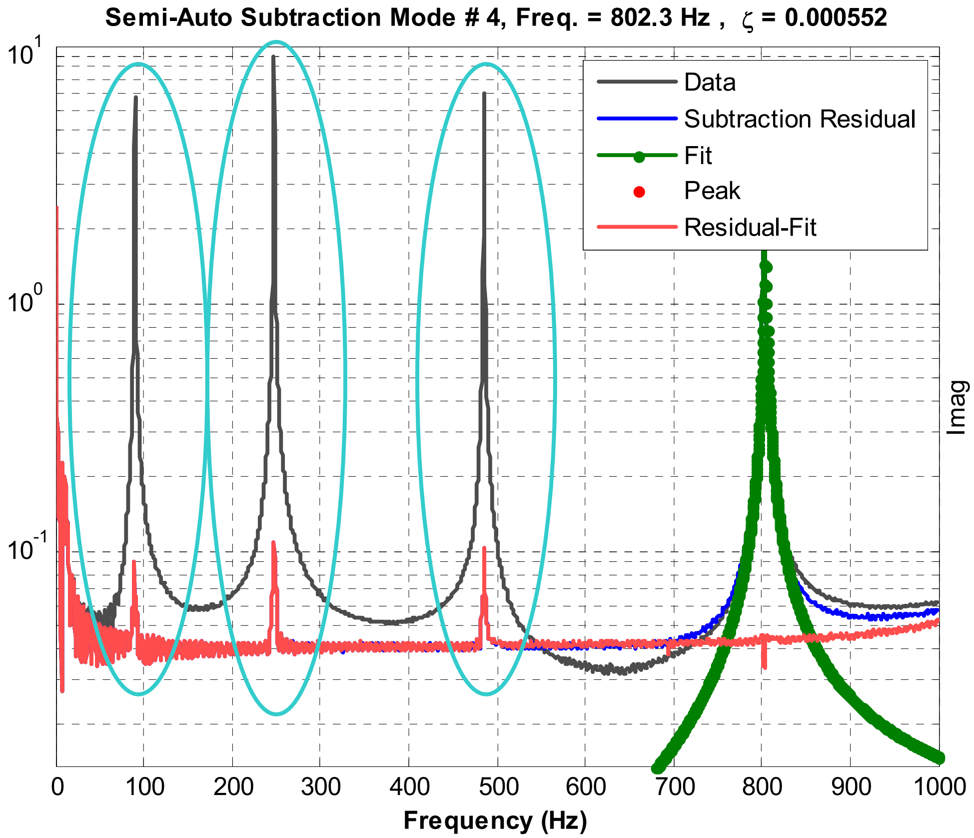
Automatic Subtraction

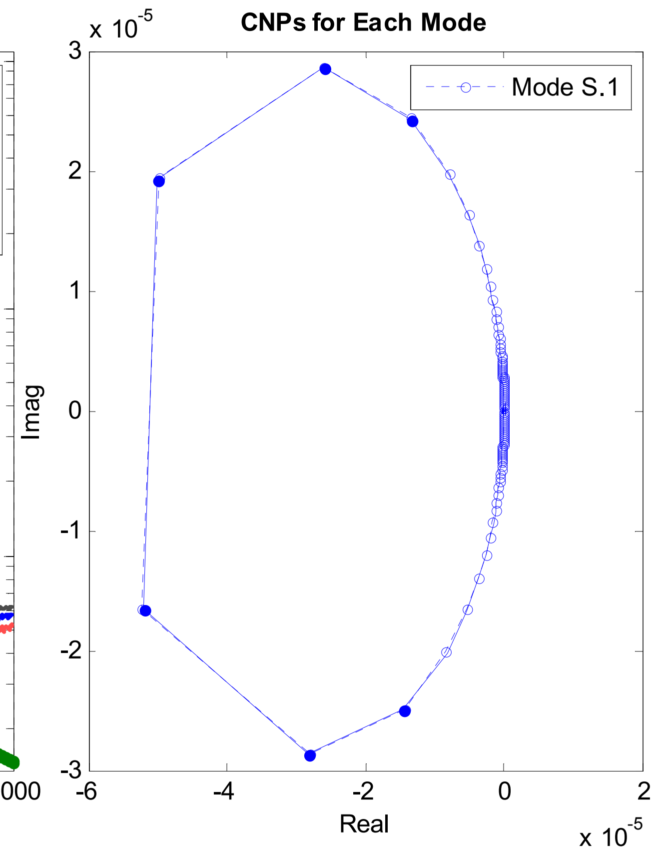
- For simple data many of the modes can be extracted automatically. Try this feature on the 17 point beam data using:
  - `AMISET.AutoSubLevel = 0.5;`
  - This tells AMI to open the measurements and subtract modes until the highest peak (versus frequency) has been reduced to 50% of the height of that in the original measurement.

```
clear all; close all;

load('ffbeam_17_Hjp')    % FRF data from SIMO test on free-free beam.
% load('ffbeam_20input_MIMO')    % FRF data from MIMO test on free-free beam.

global AMISET        % Initialize options as global variable
AMISET = AMIG_def_opts; % Set default options

% Can change specific options manually, see AMIG_def_opts.m for details
AMISET.FRF_RF = 10; %this sets minimal reduction factor, used to prompt higher order fit
AMISET.AutoSubLevel = 0.5;
AMISET.DVA = 'A'; % this tells AMI that the data is acceleration/force FRFs
AMISET.LM = 'on'; % this makes the file smaller if you save the AMI fit.

ws = 2*pi*fs;      % Convert from Hz to rad/s
ami(H,ws,AMISET)   % where H is the Nf by No FRF matrix and ws is the frequency vector in radians per second.

% Modal Data
global AMIMODES % brings AMI's variables into the workspace
wn = AMIMODES.mode_store(:,1)       % Natural frequencies, rad/s
zt = AMIMODES.mode_store(:,2)       % Damping ratios
res = -2*real(diag(conj(AMIMODES.mode_store(:,3)))*AMIMODES.A_store(:,:,1)); % Residue vector
```

Automatic Subtraction

- AMI proceeds to the point shown below before halting. Notice that three modes have already been identified, judged to be valid, and subtracted from the measurements (left three peaks) and AMI is currently considering the fourth peak.
  - As is typical, a small peak remains in the residual (blue line below) near the natural frequency of each of the identified modes.





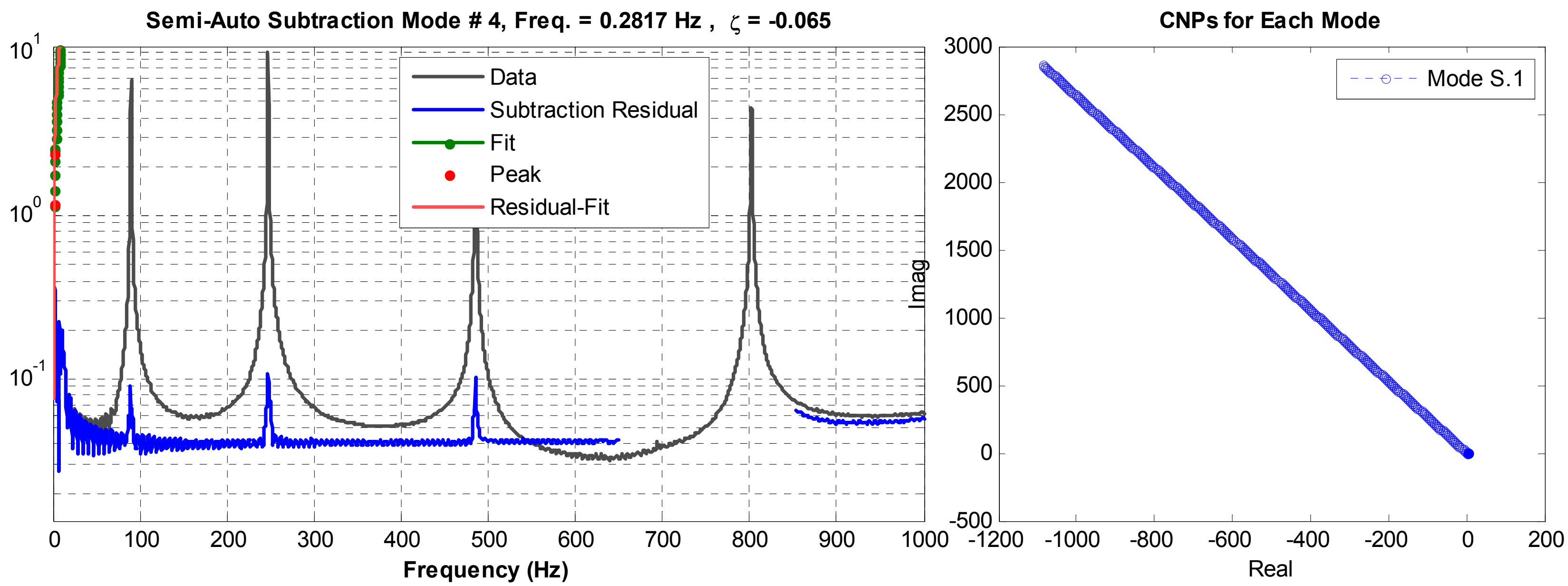
Automatic Subtraction (2)

- The command window shows that three modes have been identified at 247, 485 and then 89.9 Hz. A mode at 802 Hz has been identified at the peak in question and the algorithm asks whether to proceed.
- The user must now decide whether to:
  - keep the mode (y)
  - or zero out the data in question (z)
  - Note that there is also an option to not keep the mode (n), but in the automatic mode of operation this will cause the algorithm to return to this same mode again so this should not be used.
- If the fit looks reasonable, simply accept the mode (y). Accepted modes can be deleted from extraction set later on.

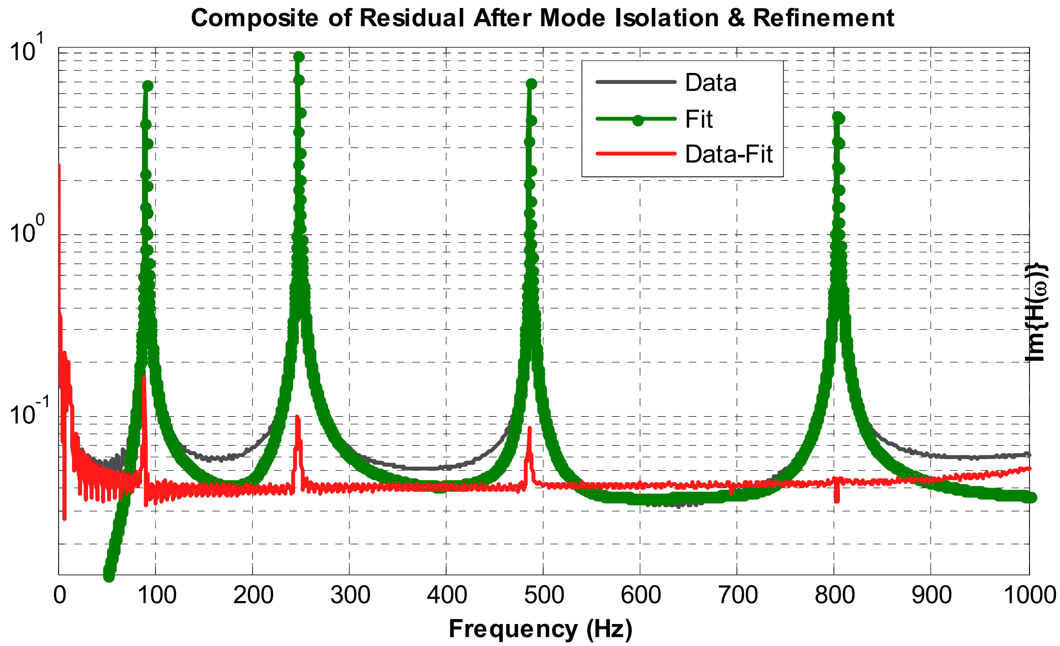
```
Mode Identification & Subtraction Stage
Mode Added: fn = 247.406                 Zeta = 0.000786873
Mode Added: fn = 485.375                 Zeta = 0.000384549
Mode Added: fn = 89.9448                 Zeta = 0.00210287
Good Modes So Far:
Mode:      fn =      zeta =    lambda =
1            247.41            0.00078687          -1.2232 + i*  1554.5
2            485.37            0.00038455          -1.1728 + i*  3049.7
3            89.945            0.0021029           -1.1884 + i*  565.14
Current Mode:
4            802.33            0.00055172          -2.7813 + i*  5041.2
RFtotal = 0.45832, RF_SDOF = 99.4289 RF_SDOF_full = 99.4289
Singular Value Ratios Sj/S1
  1   0
Keep the Last Mode (y,n or z)?
```

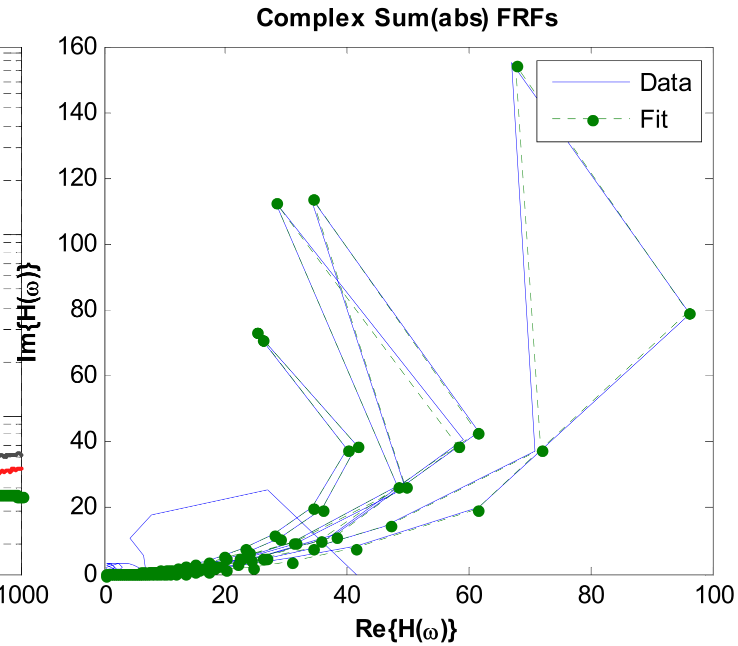
How to Run AMI in MATLAB

- To zero out data (so that it will no longer be considered during this subtraction step):
  - enter (z) in command window
  - when you are ready (e.g. after zooming in) click the zero button in the GUI (lower right) and then
  - click once on the lower and upper limits of the region that you would like to zero.
- For example, the plot below shows the result of zeroing the measurement from 650 to 850 Hz.
- The algorithm now tries to fit the noise in the measurement near 0Hz, giving a spurious mode, so it is time to end subtraction and refine the results by isolating.

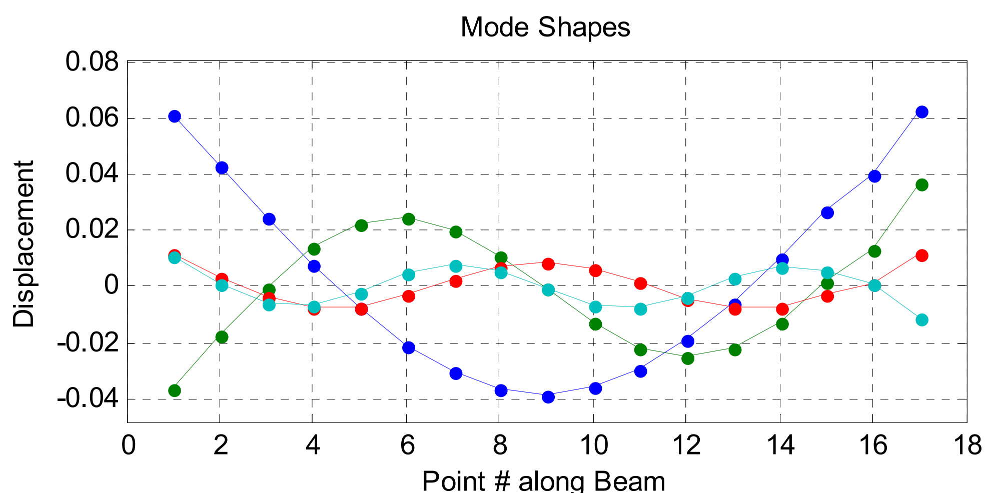


Final Results for 17-node F-F Beam



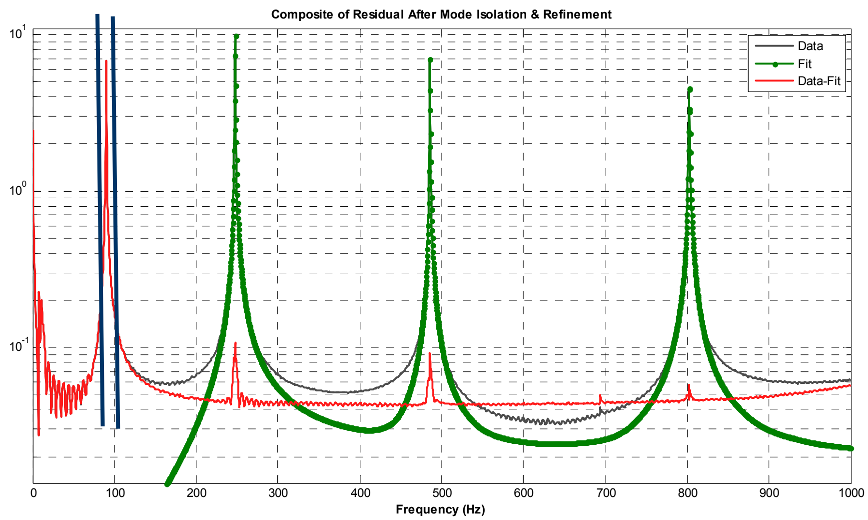


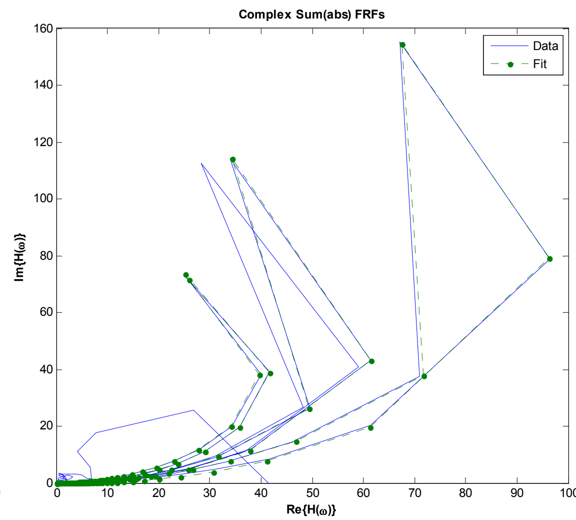
- The modal parameters are stored in a global structure called "AMIMODES"
  - **`AMIMODES.mode_store`** contains the natural frequencies and damping ratios.
  - **`AMIMODES.A_store`** contains the modal residues, which are related to the mode shapes, as explained later.



Other Options

- Suppose we wanted to remove a mode from the set. Enter (y) when prompted to discard modes, and enter [mode numbers] in list to be deleted. For example, we discard the first mode.
- Another feature is manual extraction. We will use this to recover the first mode after discarding it. After the algorithm has finished (in command window). In the GUI (Figure 1000), navigate to AMI-Menu>Manual Subtract.
- Click BandSel in lower right, and select the bandwidth of the peak to be extracted (e.g. along blue vertical lines below).





Recovering Classical Modes

- AMI Fits a model based on state space (arbitrarily damped) modes. Often we wish to recover the best fit classical, or real mode from these.
- State space and classical modes describe the FRF as follows.

State Space

$$[\mathbf{H}] = \sum_{r=1}^{N} \frac{\mathbf{A}_r}{\mathrm{i}\omega - \lambda_r} + \frac{\mathbf{A}_r^*}{\mathrm{i}\omega - \lambda_r^*}$$

```
A=AMIMODES.A_Store
% N x No x Ni
(number of modes) x
(number of outputs) x
(number of inputs)
```

Forced to be Rank 1 Matrices!!

Classical

$$[\mathbf{H}] = \sum_{r=1}^{N} \frac{\{\phi\}_r \{\phi\}_{p,r}^{\mathrm{T}}}{\omega_r^{\ 2} - \omega^2 + \mathrm{i}\omega 2\zeta_r \omega_r}$$

```
A=AMIMODES.A_Store
See next slide for best
fit classical mode
```

Recovering Classical Modes

- To recover the best fit classical (undamped) modes, where $res(:,r) = \{\phi_r\}\phi_{rp}$ use:

```
global AMIMODES % brings AMI's variables into the workspace
wn = AMIMODES.mode_store(:,1); % Natural frequencies, rad/s
zt = AMIMODES.mode_store(:,2); % Damping ratios
res = -2*real(diag(conj(AMIMODES.mode_store(:,3)))*AMIMODES.A_store(:,:,1) .');
% To retain imaginary parts (to examine modal phase colinearity), use:
Res = -2*diag(conj(AMIMODES.mode_store(:,3)))*AMIMODES.A_store(:,:,1).';
```

  - "`Res(:,r)`" can be plotted on the real-imaginary plane to see whether a real mode shape is appropriate.

Mode Shapes and Residue Vectors

- The residue vector is related to the mode shape by

$$res(:,r) = \{\phi_r\}\phi_{rp}$$

  - Where *r* is the mode index, *p* is the drive point.

- If, for example, the drive point was the first measurement point (p=1), then

$$res(1,r) = \phi_{r1}\phi_{r1} = \phi_{r1}^2, \quad then, \quad \phi_{r1} = \sqrt{res(1,r)}$$

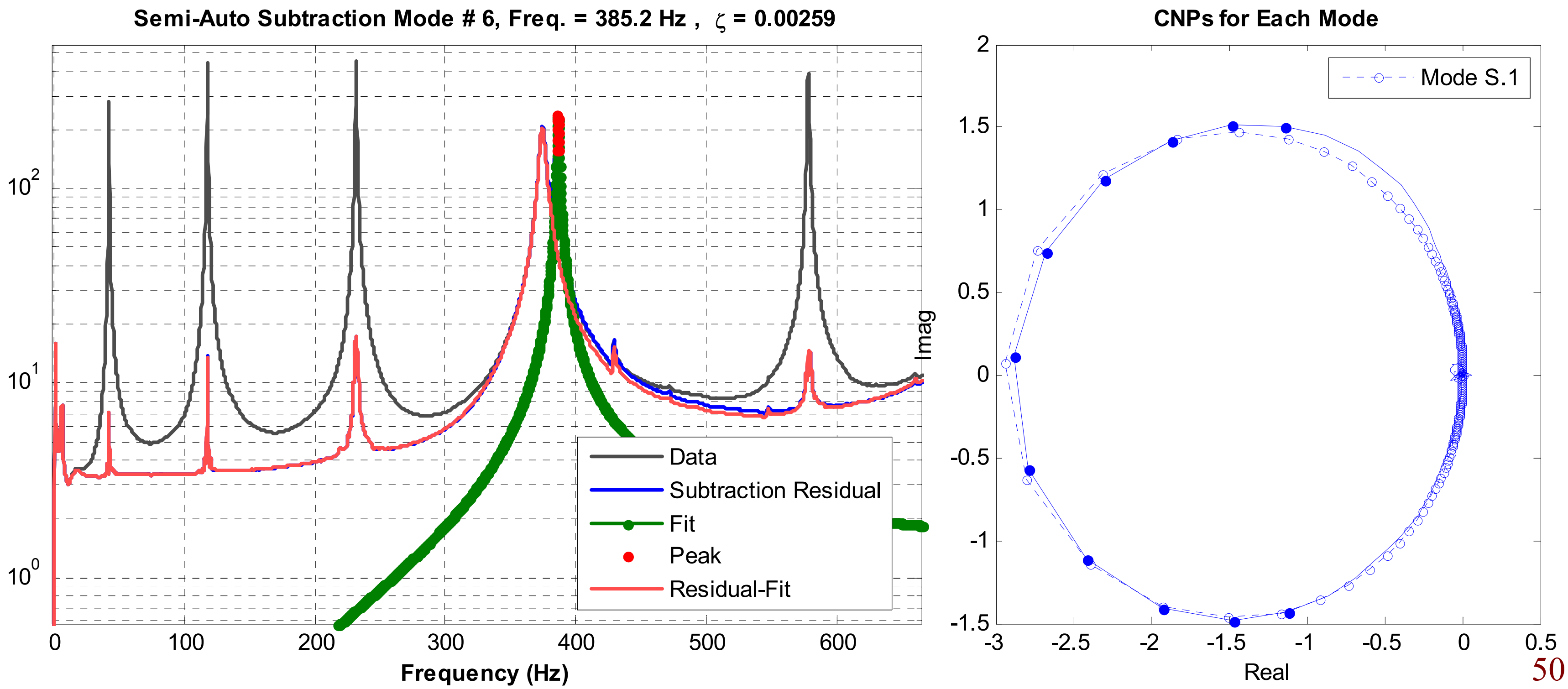
$$res(2,r) = \phi_{r2}\phi_{r1}$$

- The modal amplitude for the second point (and similarly for all others) can be computed as

$$\phi_{r2} = \frac{res(2,r)}{\phi_{r1}}$$

MIMO Data and Close Modes

- Now execute the “runAMI.m” script again but uncomment the second line to load the 20-node, MIMO data:
  - `load('ffbeam_20input_MIMO')`
- The algorithm identifies five modes automatically, but the sixth appears to be a pair of close modes so the user is prompted.



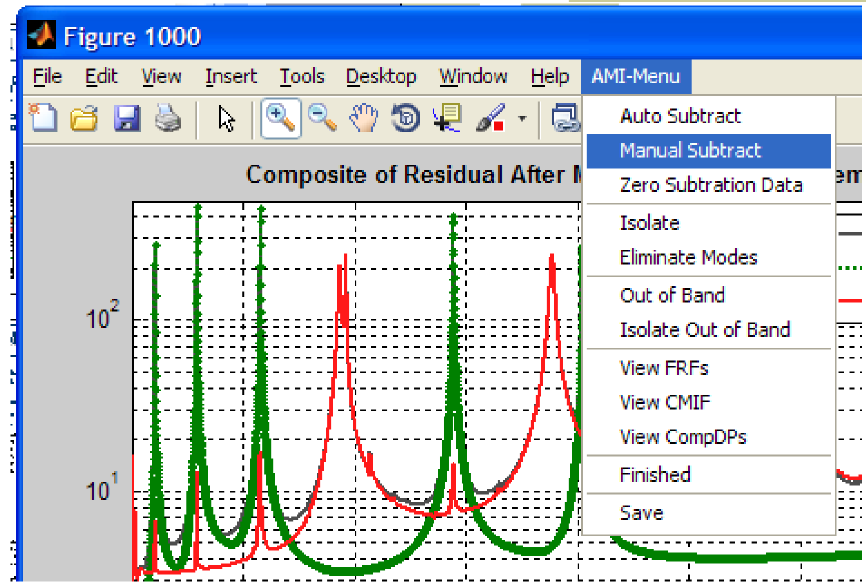
Close Modes

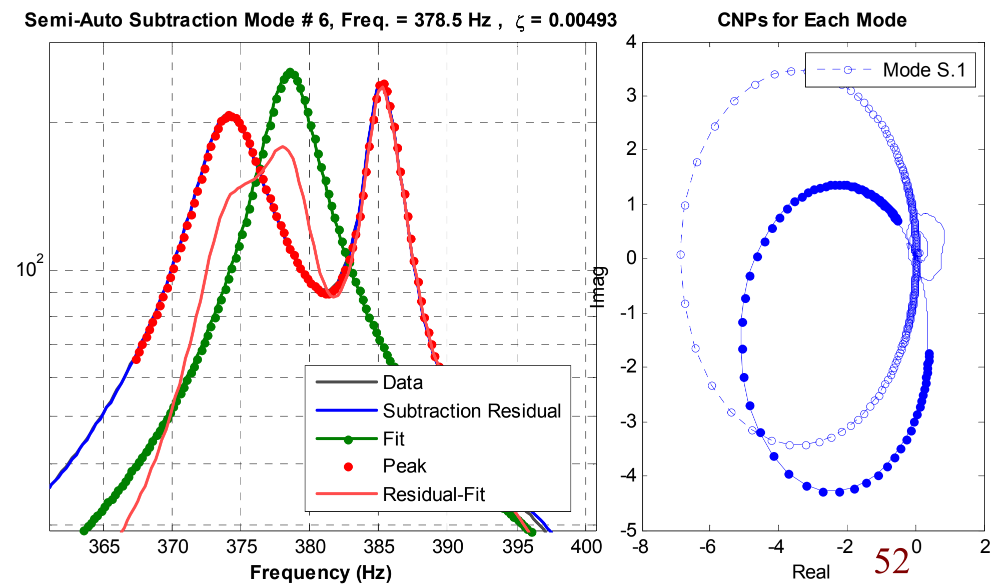
- The algorithm has halted because it detects a second singular value of 0.208, suggesting that the FRF at this peak contains a second shape that is 20% as important as the first.
  - The user is asked whether to fit two modes ot this peak.
- Visual inspection reveals that this is due to a nearby mode, so the mode could be accepted and we could continue.
- However, instead let's answer no, end subtraction and try fitting two modes simultaneously.
- Answer as follows:
- `Do you want to try a higher order fit? (y,n) n`
- `Keep the Last Mode (y,n or z)? n`
- `Continue Subtraction? n`
- `Do you want to discard any modes (y/n)? n`
- `Look for More Modes? n`

```
Mode Identification & Subtraction Stage
Mode Added: fn = 231.489                          Zeta = 0.00166216
Mode Added: fn = 117.661                          Zeta = 0.00210203
Mode Added: fn = 577.775                          Zeta = 0.0010424
Mode Added: fn = 42.4222                          Zeta = 0.00396566
Mode Added: fn = 809.056                          Zeta = 0.000831241
Good Modes So Far:
Mode:        fn =        zeta =       lambda =
1              231.49    0.0016622    -2.4176 + i*  1454.5
2              117.66    0.002102      -1.554 + i*  739.29
3              577.78    0.0010424    -3.7842 + i*  3630.3
4              42.422    0.0039657     -1.057 + i*  266.54
5              809.06    0.00083124   -4.2256 + i*  5083.4
Current Mode:
6              385.22    0.0025908    -6.2708 + i*  2420.4
Current mode has not reduced the FRF Substantially,
or a second singular value may be significant.
     RFtotal = 0.54114, RF_SDOF = 3.9816 RF_SDOF_full = 4.4152
     Singular Value Ratios Sj/S1
     1     0.20802
Do you want to try a higher order fit? (y,n)
```

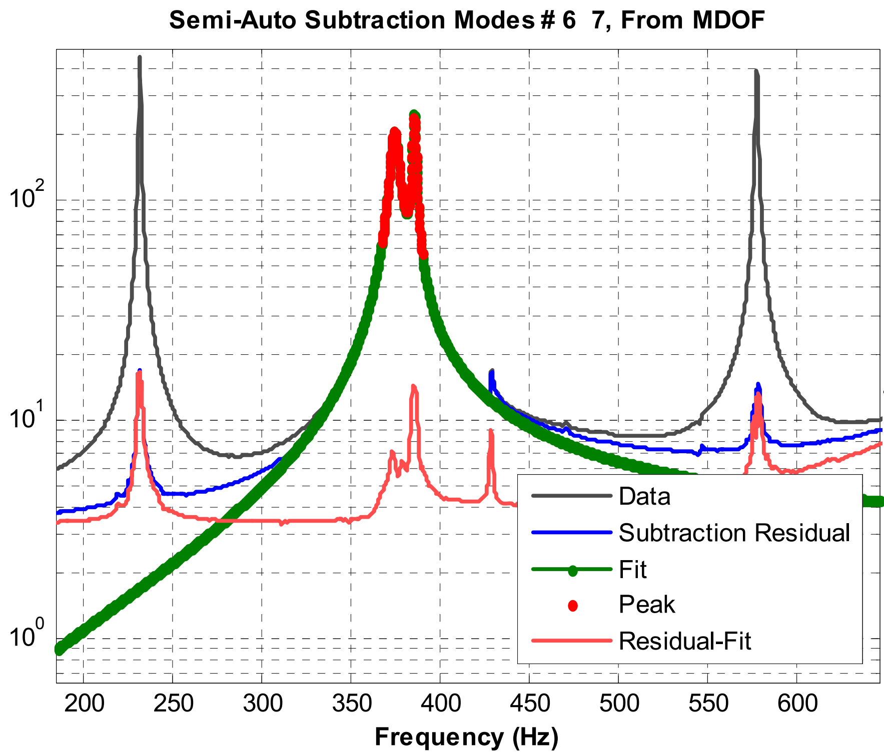
Manual Subtraction and Close Modes

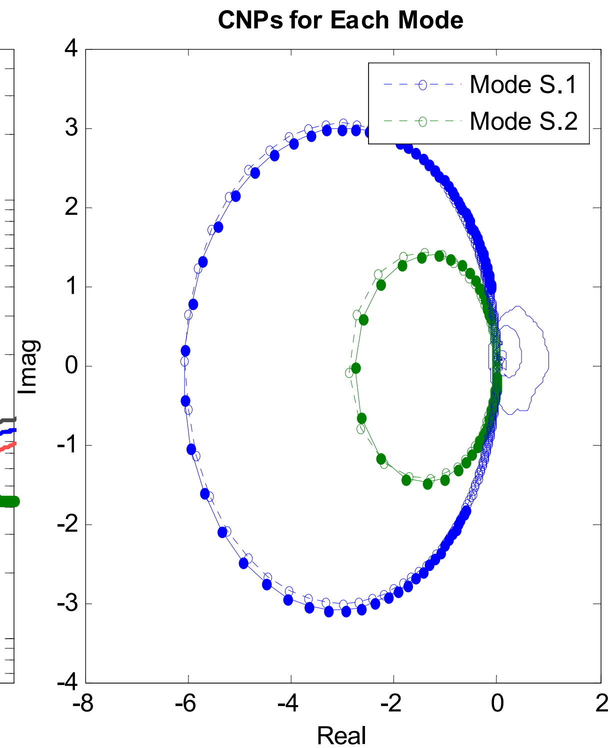
- Select "Manual Subtract" and select a band of frequencies from 370 to 390 Hz.
- AMI has now clearly fit one mode where two are actually present.
- This is evidenced visually and by the singular value ratios: [1, 0.53].
- The fit improves substantially with two modes, as seen on the next slide.





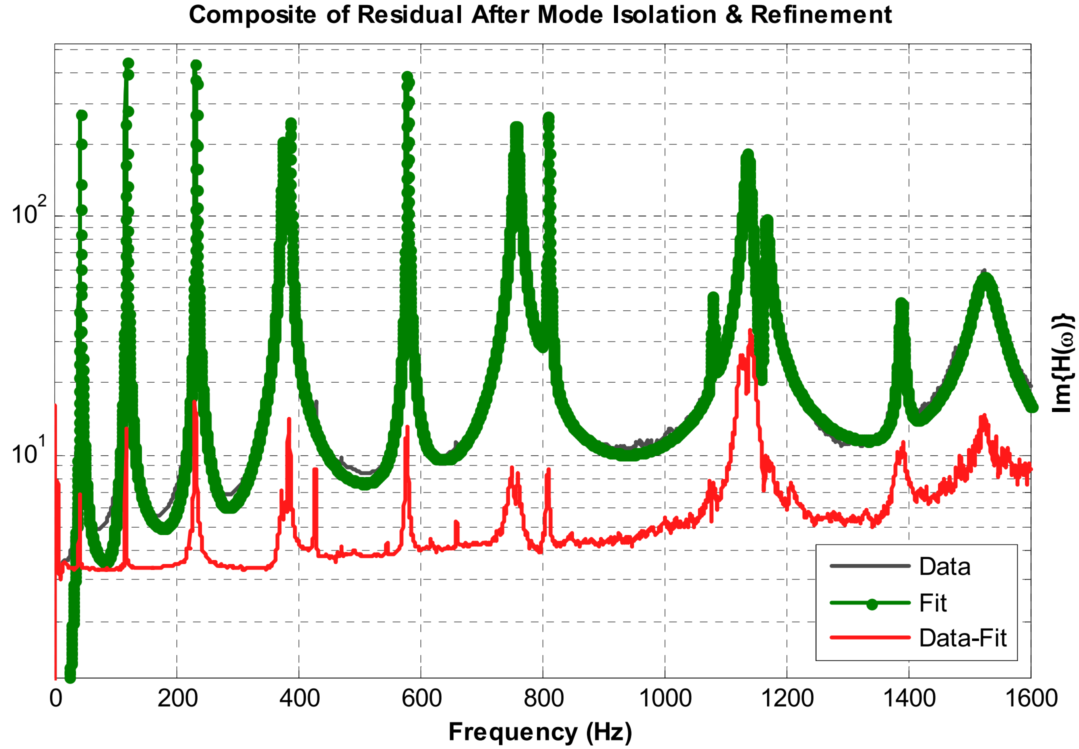
Multi-Mode Fit with AMI

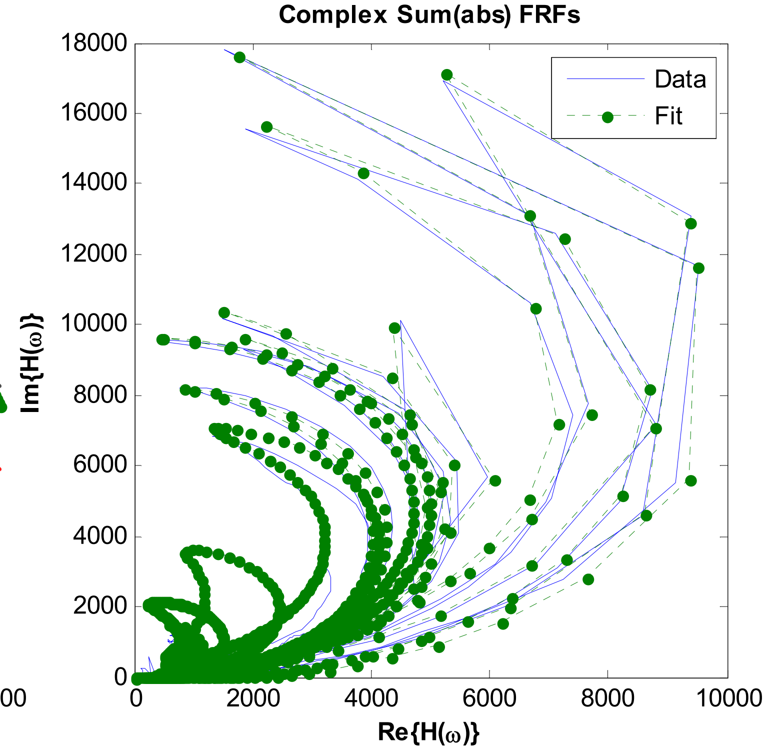




- These modes have been identified as a pair of close modes and will be identified simultaneously using the Frequency Domain Subspace Algorithm during any subsequent isolations stages.
- Now, we can end manual subtraction and return to the automatic mode to extract the rest of the modes.

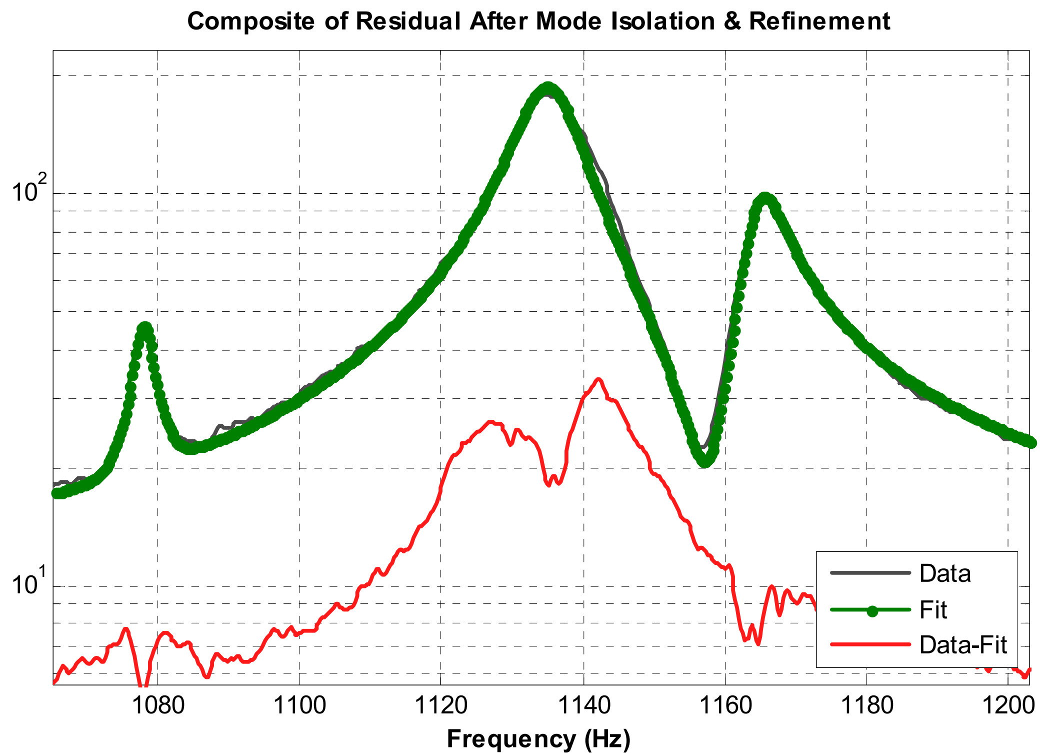
20 Node Beam with MIMO: Results

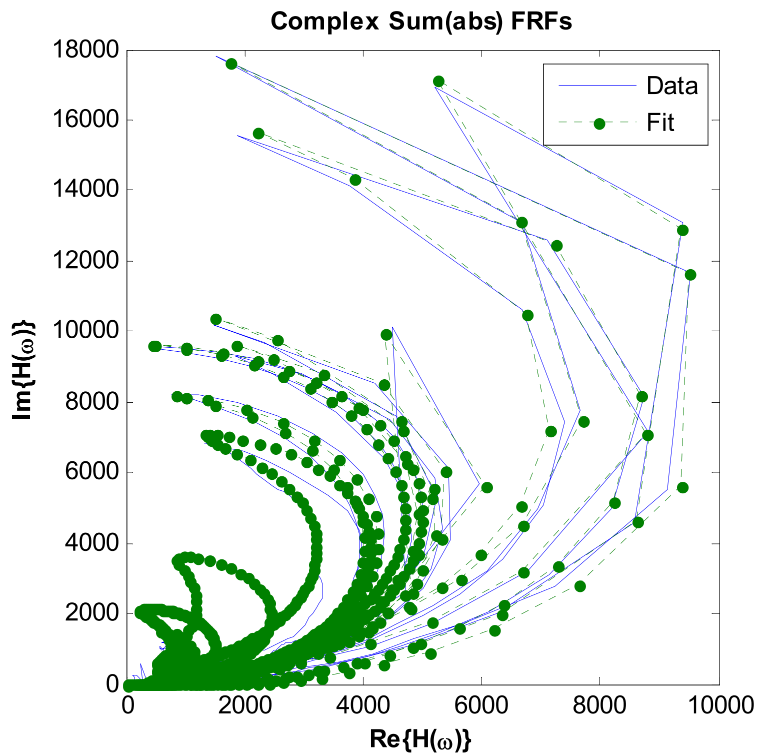




- The curve fit seems to follow the measurements really well, although there is one area near 1140 Hz that doesn't seem to be fit very well.
- What should we do?

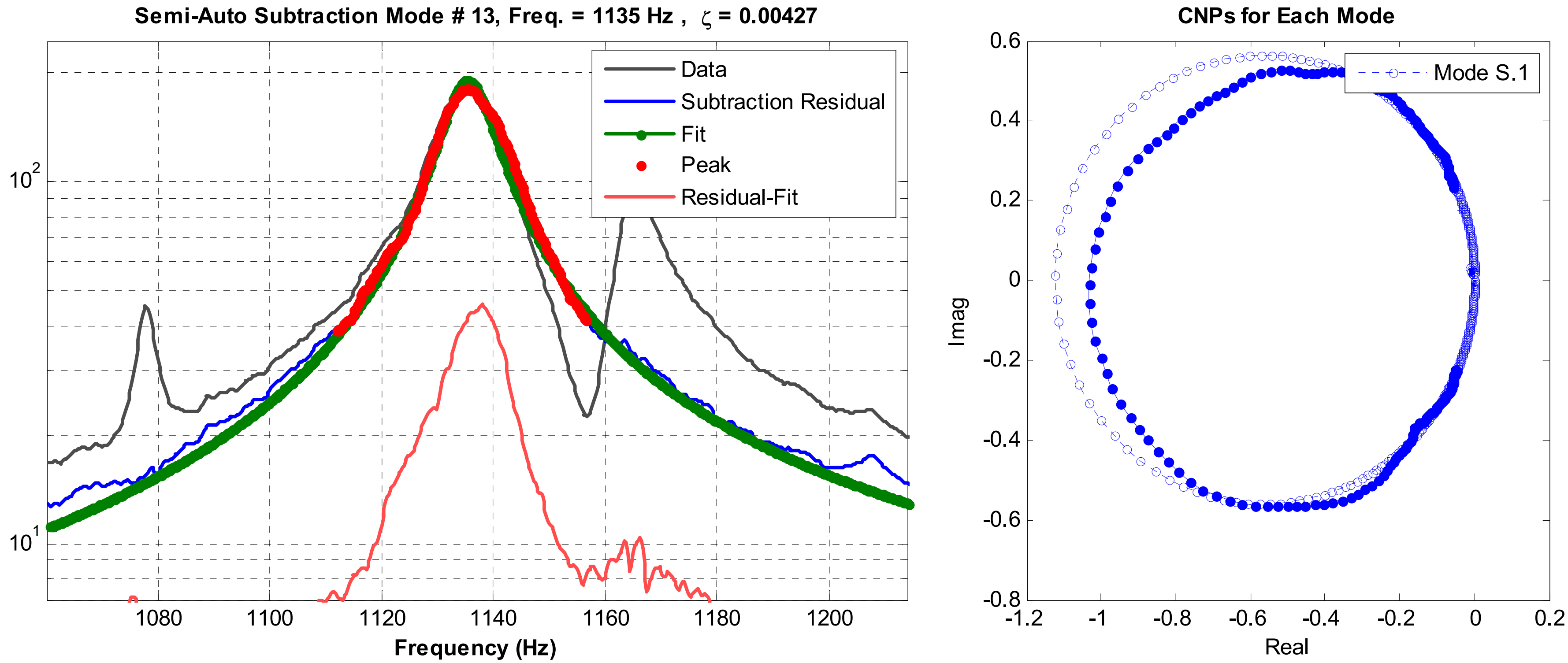
Manual Refinement





- The residual is large near 1140 Hz – could there be a second mode in there?
- Try eliminating that mode and fitting two modes there.
  - AMI Menu > Eliminate Modes >
  - (command window) >> y
  - (command window) >> [10]

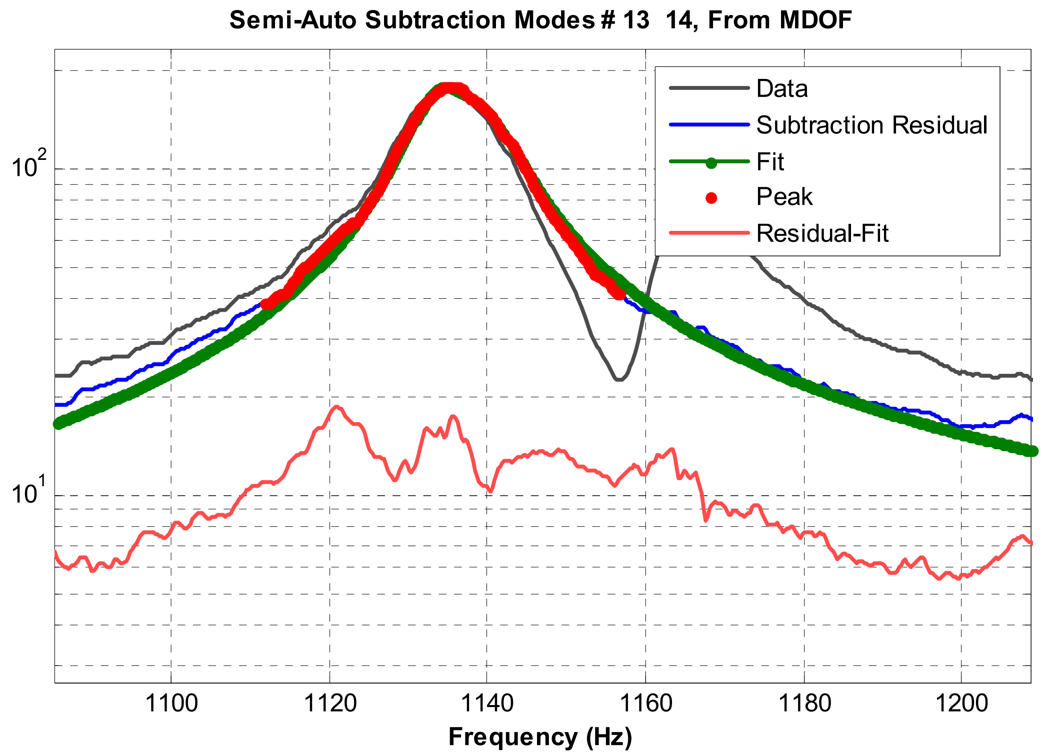
Manual Refinement

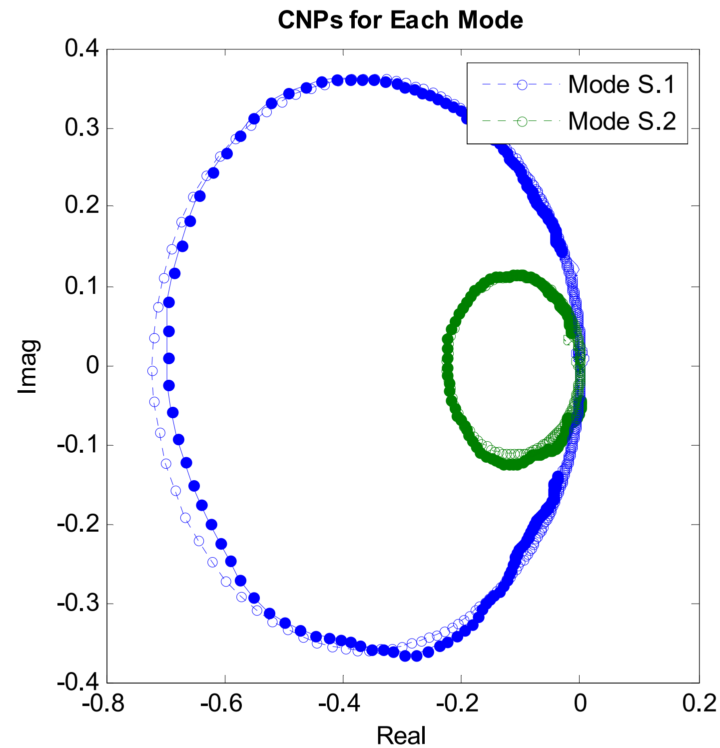


- Fitting a wider band manually, the curve fit is not so satisfying, but the singular value ratio is still very small ([1, 0.023]), so there is little evidence of a second mode near this peak.
- Try a 2 DOF curve fit just to see.

```
13            1135.2    0.0042743    -30.487 + i*  7132.6
Current mode has not reduced the FRF Substantially,
or a second singular value may be significant.
     RFtotal = 0.40314, RF_SDOF = 3.9528 RF_SDOF_full = 3.958
     Singular Value Ratios Sj/S1
     1    0.023712
Do you want to try a higher order fit? (y,n)
```

Manual Refinement

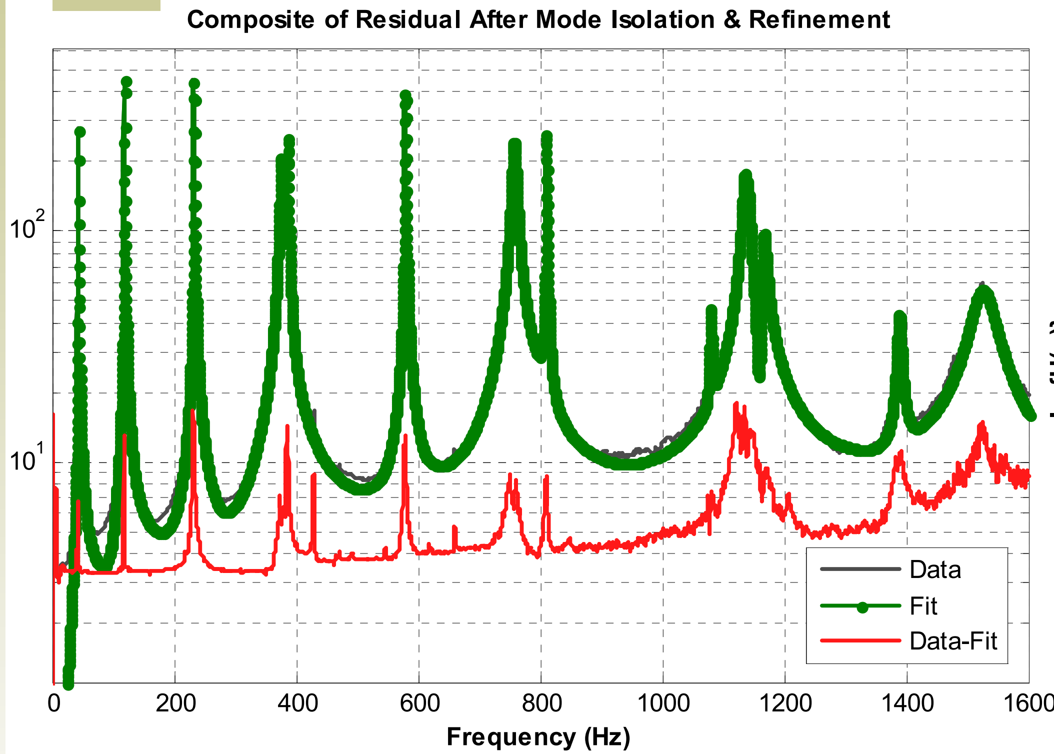


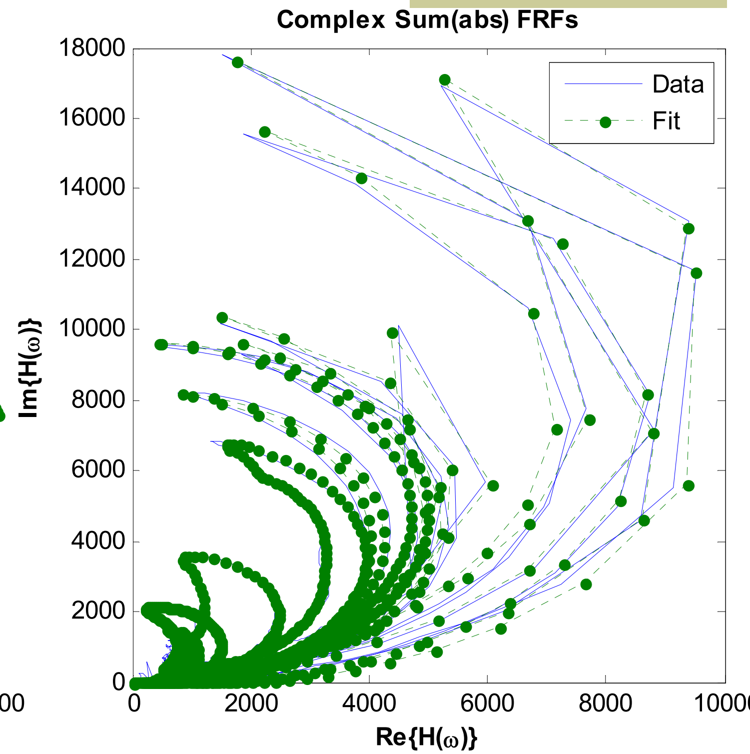


- Surprisingly, the 2-mode fit agrees quite well with the measurements and the two modes have distinct frequencies, so this second mode should probably be retained.

```
Do you want to try a higher order fit? (y,n) y
Input Order to try (2 or 3 typical)?2
MDOF Mode Parameters, 2 DOF:
1               1133.8    0.0039669    -28.259 + i*  7123.7
2               1139.4    0.0042993    -30.779 + i*    7159
(RFtotal = 0.40314, RF_MDOF = 3.5335 RF_SDOF_full = 3.958)
```

Final Curve Fit, 20-node beam.

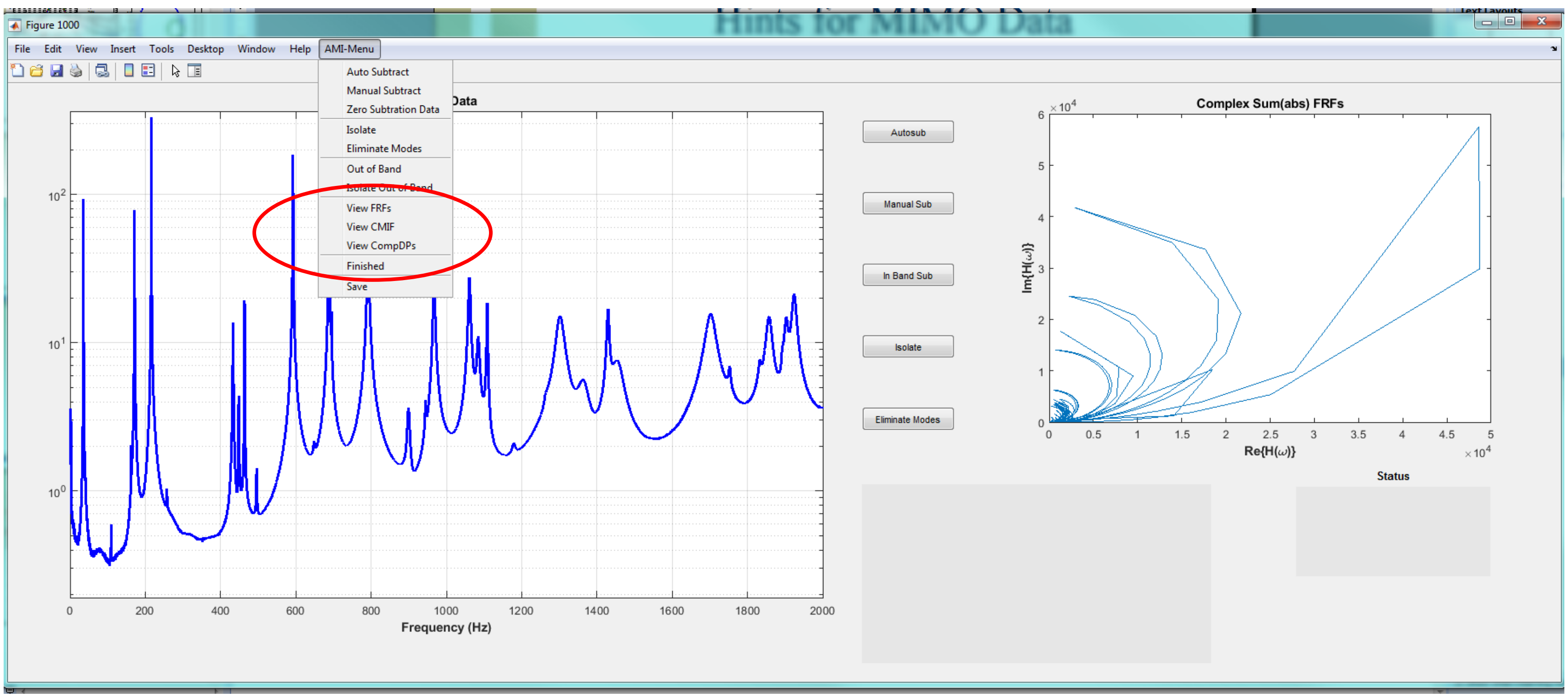




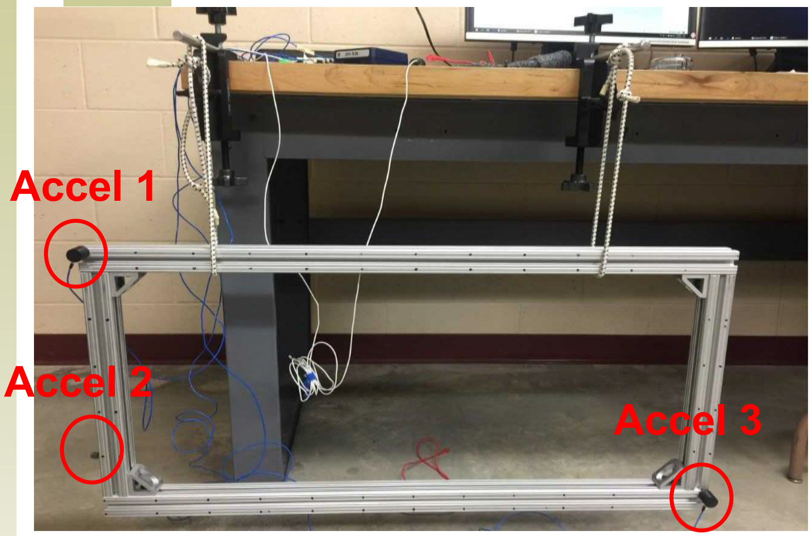

```
Mode Isolation Complete - Minimum Change Reached in 7 iterations
Mode:   fn =          zeta =       lambda =               norm(A)          group #
1       42.422        0.0039343    -1.0487 + i*  266.55   0.031302         4
2       117.66        0.0021023    -1.5542 + i*  739.28   0.0093763        2
3       231.49        0.0016623    -2.4179 + i*  1454.5   0.0035802        1
4       374.23        0.0065874    -15.489 + i*  2351.3   0.0041205        6
5       385.28        0.0024386    -5.9033 + i*  2420.8   0.0016997        6
6       577.78        0.001042     -3.7827 + i*  3630.3   0.00080465        3
7       755.49        0.0049779    -23.629 + i*  4746.8   0.0018366        8
8       809.06        0.0008313    -4.2259 + i*  5083.5   0.00031604        5
9       1077.8        0.0013728    -9.2967 + i*  6772.3   7.4327e-005       10
10      1133.7        0.0039052    -27.819 + i*  7123.4   0.00061011        12
11      1139.2        0.0045046    -32.244 + i*  7157.8   0.00040257        12
...
```

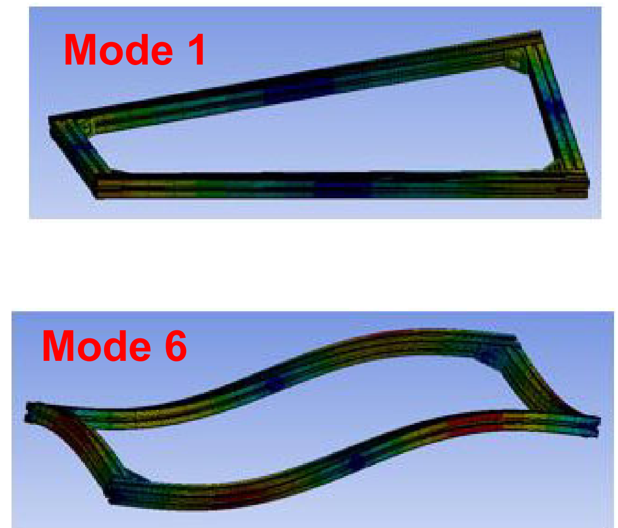
Hints for MIMO Data

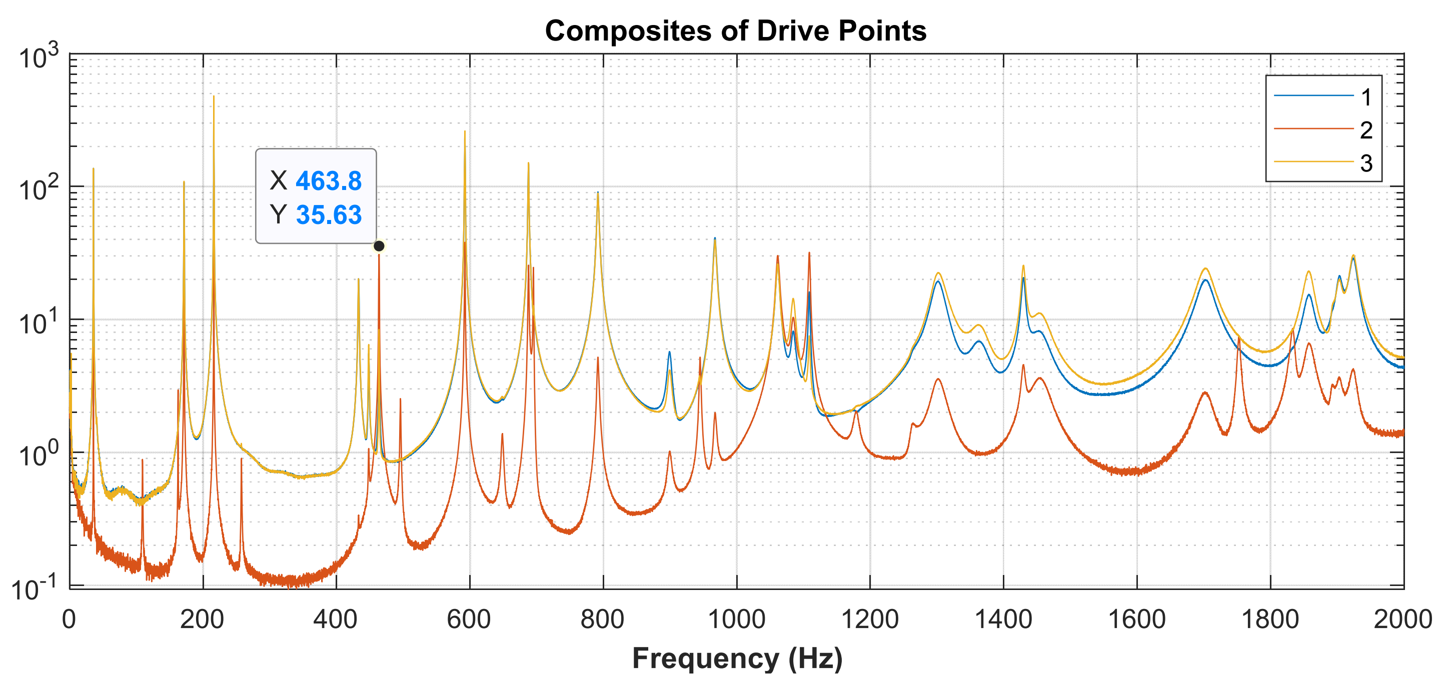
- A few plots that are easily accessible in the AMI Menu can help to decipher MIMO data:
  - View CMIF – Mode Indicator Function
  - View CompDPs – Composite of all Drive Point Accelerometers.



Example: Box Frame



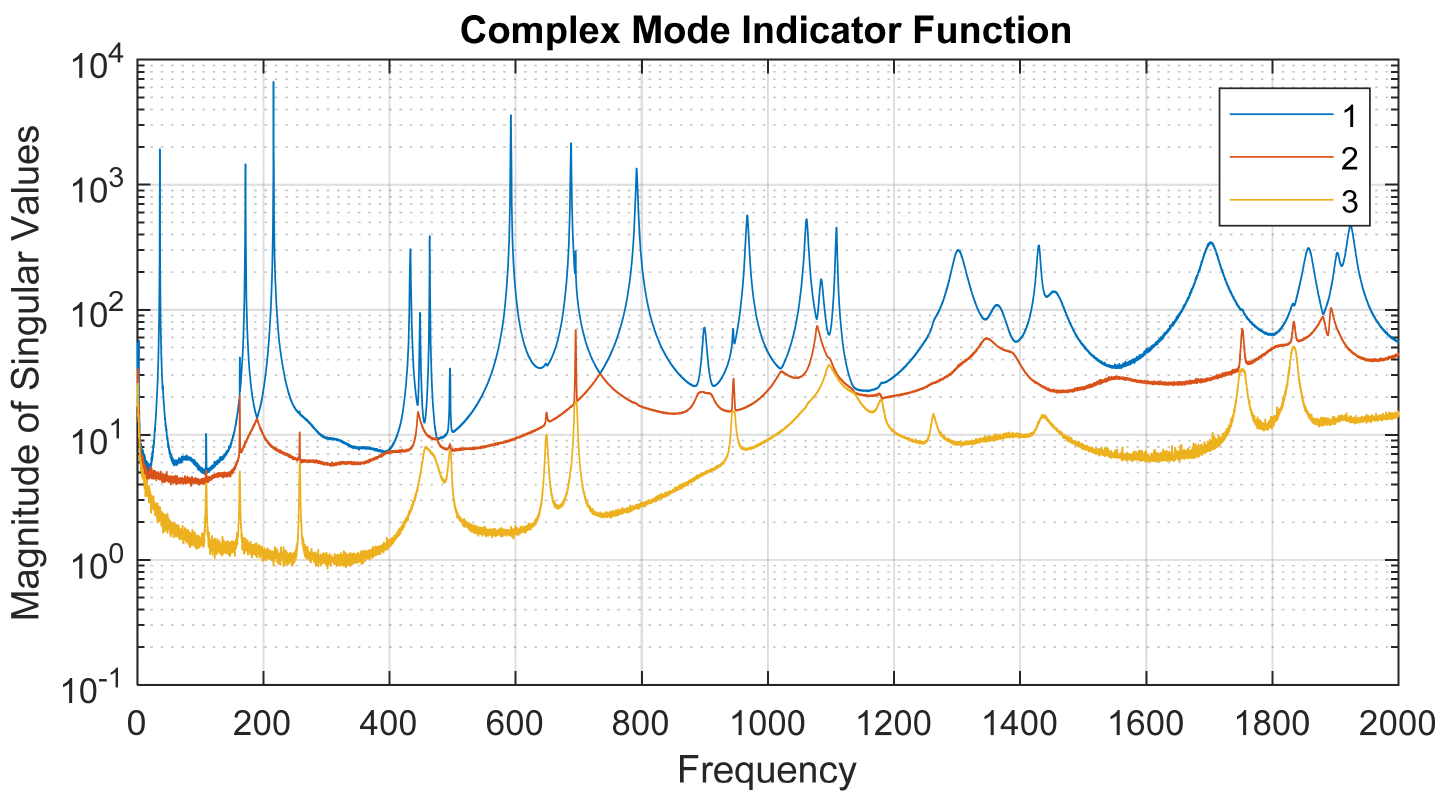




The composites of the drive points shows that Modes 1-5 are dominated by out-of-plane motion because their response is strong in Accels 1 and 3.

Mode 6 at 438 Hz is the first mode to be dominated by in-plane motion, as evidenced by its large motion in Accel. 2.

CMIF from AMI

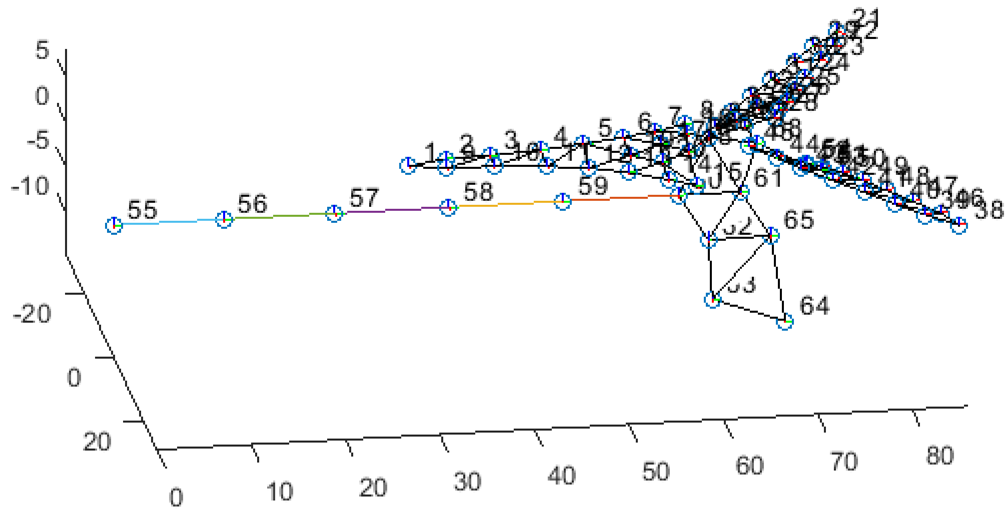


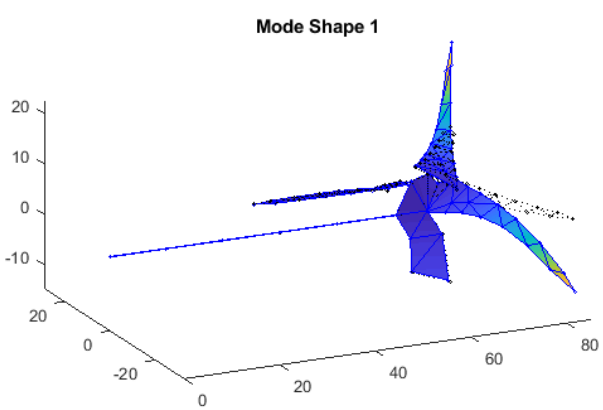
- The CMIF is used to detect modes with close natural frequencies. At each frequency the FRF matrix is decomposed into N_dp shapes, where N_dp is the number of drive points (accelerometers in this case). The strength of those shapes is then plotted.
- If only one mode is present, the blue line will show a peak and all other lines will be noise. This is the case for all modes here.
- At 1100 Hz we can see that the modes are close enough together that a second peak (red line) is significant between modes. See the lecture notes from class for a case of a plate with repeated natural frequencies.

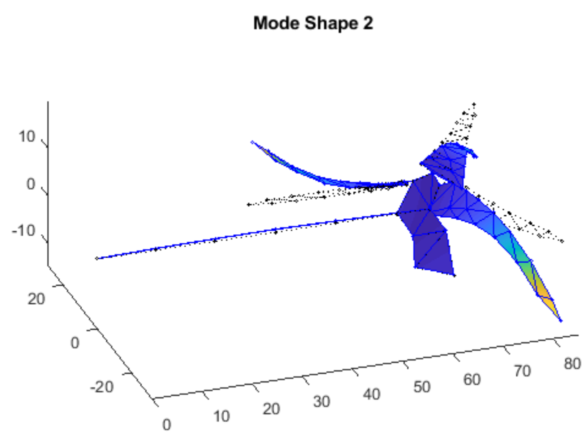
Plotting Modes with "uff_geo"

- Go to EMAfun\uff_geo and run: "AAA_ModalAnalysisPlotGeo_TestScript.m"
- If your path is set correctly this will show the mode shapes of a wind turbine.
- You should be able to use similar commands to plot the mode shapes of your structure if you use the following steps:
  - Save your geometry to a UFF file. Select your geometry from the "project tree" and right click and "save to file". Select "uff" as the type and give it a name.
  - These commands should then create a plot of your geometry:
  - geo=uff_geo('YourGeometry.uff');
  - figure(1)
  - plot_geo(geo,'labels','cs');
  - axis equal;
  - axis tight;

Sample Output from uff_geo







- This is the undeformed geometry for the wind turbine. You can see three blades and a tail (pointed downwards)
- After picking a few peaks we can plot their mode shapes