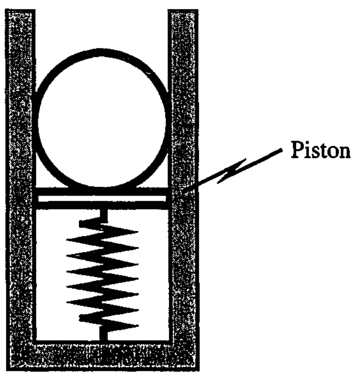
It is instructive to observe that if $K < 0$, then the homogeneous solution to the equation of motion is

$$q = B_1 \exp\left(\sqrt{\frac{-K}{M}}t\right) + B_2 \exp\left(-\sqrt{\frac{-K}{M}}t\right) \tag{2.2.12}$$

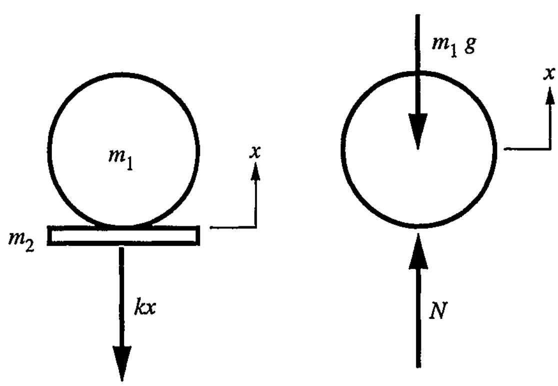
Thus, one of the solutions grows with increasing time. We say that this is a *divergence instability*. The small displacement approximations leading to linearized equations of motion are valid only for a very short time when a system is unstable. One indication of this loss of validity is the fact that the response in eq. (2.2.12) does not conserve energy. We will encounter another type of instability in Chapter 11, where we study time-dependent systems.

EXAMPLE 2.5

The 200 gram sphere is not attached to the 50 gram piston. The stiffness of the spring is 1200 N/m. The spring is held 80 mm below the static equilibrium position, and then released. Determine the position of the piston above the equilibrium position, and the corresponding elapsed time, at which the sphere ceases to be in contact with the piston.



Solution This exercise is intended to bring out the way in which static and dynamic forces might occur in a study, as well as to highlight interpretation of harmonic response. We begin by drawing two free body diagrams. The first, which we will use to derive the equation of motion, considers the sphere and the platform as a system, so that the contact force exerted between these bodies is an internal force that is not considered.



We consider the system to move away from its static equilibrium position a distance x upward, which serves as the generalized coordinate. Hence, the spring force acts downward. The gravity force has a static effect, which is irrelevant to the equation of motion because x is measured relative the static equilibrium position. Hence, the equation of motion resulting from the first free body diagram is

$$m\ddot{x} + kx = 0, \quad m = m_1 + m_2 = 0.25 \text{ kg}$$

The second free body diagram isolates the ball because we seek the conditions under which the contact force N applied by the platform becomes zero. We show the gravity force in this free body diagram because the weight affects the normal force N, and the condition we seek is $N = 0$. From the second free body diagram, we find that

$$N - m_1 g = m_1 \ddot{x}$$

Hence, $N = 0$ is marked by $\ddot{x} = -g$.

The idea now is to find the response corresponding to the given initial conditions, and then to determine when the acceleration condition $\ddot{x} = -g$ occurs. We write the equation of motion as

$$\ddot{x} + \omega_{\text{nat}}^2 x = 0$$

where the natural frequency is

$$\omega_{\text{nat}} = \left(\frac{k}{m}\right)^{1/2} = 69.28 \text{ rad/s}$$

The given initial conditions indicate that $x = -0.08$ m and $\dot{x} = 0$ at $t = 0$. The response matching these initial conditions is

$$x = -0.08\cos(\omega_{\text{nat}} t)$$

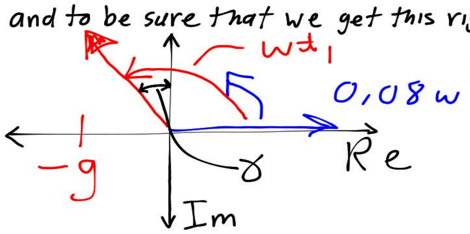
or, equivalently,

$x(t) = \text{Re}(-0.08\, e^{i\omega t})$

We want to know the first instant at which $\ddot{x} = -g$, so we write

$\ddot{x}(t) = \text{Re}(0.08\,\omega^2 e^{i\omega t})$

and to be sure that we get this right we'll use a sketch:



$0,08\omega^2 = 383,98 \text{ m/s}^2$

Using the triangle above to find γ:

$$\gamma = \sin^{-1}\left(\frac{9.81 \text{ m/s}^2}{383.98 \text{ m/s}^2}\right) = 0.0255 \text{ rad} = 1.46^\circ$$

So, for this to occur, the vector must sweep through an angle: $\pi/2 + \gamma$

so $\omega t_1 = \pi/2 + \gamma \longrightarrow t_1 = (1.57 + 0.0255)/69.28 = 23.04$ ms